Введение

Настоящий сборник задач предназначен для самостоятельной работы студентов при подготовке к контрольным, лабораторным работам, зачёту по курсу "**Численные методы решения уравнений математической физики и химии**". Данный курс относится к разделу специальных дисциплин и читается студентам факультета кибернетики химико-технологических процессов Российского химико-технологического университета им. Д. И. Менделеева. Целью курса является изучение методов численного решения дифференциальных уравнений, входящих в состав математических моделей различных физико-химических и химико-технологических процессов, на основе разностных схем.

Сборник задач содержит следующие типы заданий, изучаемых в указанном специальном курсе:

1. Определение типа уравнения.
2. Определение порядка аппроксимации разностной схемы.
3. Исследование устойчивости разностной схемы методом гармоник.
4. Решение дифференциальных уравнений различного типа с помощью явной разностной схемы.
5. Решение одномерных дифференциальных уравнений параболического типа с помощью неявной разностной схемы (метод прогонки).
6. Решение одномерных дифференциальных уравнений параболического типа с помощью схемы Кранка–Николсона (метод прогонки).
7. Решение обыкновенных дифференциальных уравнений 2-го порядка методом прогонки.
8. Решение многомерных дифференциальных уравнений параболического типа с помощью схемы расщепления.
9. Решение многомерных дифференциальных уравнений параболического типа с помощью схемы переменных направлений.

10. Решение многомерных дифференциальных уравнений параболического типа с помощью схемы предиктор-корректор.

11. Решение многомерных дифференциальных уравнений в частных производных 1-го порядка с помощью явной разностной схемы.

12. Решение многомерных дифференциальных уравнений в частных производных 1-го порядка с помощью схемы расщепления.

13. Решение многомерных дифференциальных уравнений в частных производных 1-го порядка с помощью схемы переменных направлений.

14. Решение обыкновенных дифференциальных уравнений 1-го порядка методом Рунге-Кутта.

15. Обезразмеривание переменных в уравнениях математических моделей.

16. Запись неявной разностной схемы для системы дифференциальных уравнений.

17. Выбор метода решения для обыкновенных дифференциальных уравнений 2-го порядка.

18. Решение обыкновенных дифференциальных уравнений 2-го порядка методом простой итерации.

19. Решение обыкновенных дифференциальных уравнений 2-го порядка методом установления с использованием неявной разностной схемы.

20. Решение обыкновенных дифференциальных уравнений 2-го порядка методом установления с использованием схемы Кранка–Николсона.

21. Решение дифференциальных уравнений эллиптического типа методом простой итерации.

22. Решение дифференциальных уравнений эллиптического типа методом установления с использованием схемы расщепления.

23. Решение дифференциальных уравнений эллиптического типа методом установления с использованием схемы переменных направлений.

24. Построение тестовой задачи для дифференциального уравнения.

25. Построение тестовой задачи для системы дифференциальных уравнений.

1. Определить тип уравнения при k > 0, k = 0, k < 0.

1.1. $$2\frac{\partial^2 u}{\partial x^2} + k\frac{\partial^2 u}{\partial y^2} = 0$$

1.2. $$\frac{\partial u}{\partial t} + 3\frac{\partial u}{\partial x} = k\frac{\partial^2 u}{\partial y^2}$$

1.3. $$k\frac{\partial^2 u}{\partial x^2} + 5\frac{\partial^2 u}{\partial y^2} = 6\frac{\partial u}{\partial x}$$

1.4. $$5\frac{\partial^2 u}{\partial x^2} + k\frac{\partial^2 u}{\partial y^2} = 8\frac{\partial u}{\partial x}$$

1.5. $$k\frac{\partial u}{\partial t} = 2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

1.6. $$\frac{\partial u}{\partial t} = 2k\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

1.7. $$k\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial y^2} + 8\frac{\partial^2 u}{\partial z^2} = 0$$

1.8. $$\frac{\partial^2 u}{\partial x^2} - 2k\frac{\partial^2 u}{\partial y^2} + 8\frac{\partial^2 u}{\partial z^2} = 0$$

1.9. $$k\frac{\partial u}{\partial t} + 7\frac{\partial u}{\partial x} = 4\frac{\partial^2 u}{\partial x^2} + x$$

2. Определить порядок аппроксимации разностной схемы.

2.1. $\dfrac{u_j^{n+1}-u_j^n}{\Delta t}-0.1\dfrac{u_{j+1}^n-u_j^n}{h}=(j-1)h$

2.2. $\dfrac{u_j^{n+1}-u_j^n}{\Delta t}+3\dfrac{u_j^n-u_{j-1}^n}{h}=3n\Delta t$

2.3. $\dfrac{u_j^{n+1}-u_j^n}{\Delta t}+5\dfrac{u_{j+1}^n-u_{j-1}^n}{2h}=e^{(j-1)h}$

2.4. $\dfrac{u_j^{n+1}-u_j^n}{\Delta t}=2\dfrac{u_{j+1}^n-2u_j^n+u_{j-1}^n}{h^2}$

2.5. $\dfrac{u_j^{n+1}-u_j^n}{\Delta t}-0.1\dfrac{u_{j+1}^n-u_j^n}{h}=2\dfrac{u_{j+1}^n-2u_j^n+u_{j-1}^n}{h^2}+0.2(j-1)h$

2.6. $\dfrac{u_j^{n+1}-u_j^n}{\Delta t}+3\dfrac{u_j^n-u_{j-1}^n}{h}=7\dfrac{u_{j+1}^n-2u_j^n+u_{j-1}^n}{h^2}+5e^{n\Delta t}$

2.7. $\dfrac{u_j^{n+1}-u_j^n}{\Delta t}+\dfrac{u_{j+1}^n-u_{j-1}^n}{2h}=2\dfrac{u_{j+1}^n-2u_j^n+u_{j-1}^n}{h^2}+n\Delta t(j-1)h$

2.8. $\dfrac{u_j^{n+1}-u_j^n}{\Delta t}=\dfrac{1}{2}\dfrac{u_{j+1}^n-2u_j^n+u_{j-1}^n}{h^2}+\dfrac{1}{2}\dfrac{u_{j+1}^{n+1}-2u_j^{n+1}+u_{j-1}^{n+1}}{h^2}$

2.9. $\dfrac{u_j^{n+1}-u_j^n}{\Delta t}-0.6\dfrac{u_{j+1}^{n+1}-u_j^{n+1}}{h}=0.2\,(j-1)h$

2.10. $\dfrac{u_j^{n+1}-u_j^n}{\Delta t}-0.3\dfrac{u_{j+1}^{n+1}-u_{j-1}^{n+1}}{2h}=0.4\dfrac{u_{j+1}^{n+1}-2u_j^{n+1}+u_{j-1}^{n+1}}{h^2}-0.8n\Delta t$

2.11. $\dfrac{u_j^{n+1}-u_j^n}{\Delta t}+5\dfrac{u_j^{n+1}-u_{j-1}^{n+1}}{h}=2\dfrac{u_{j+1}^{n+1}-2u_j^{n+1}+u_{j-1}^{n+1}}{h^2}+3e^{(j-1)h}$

3. Исследовать устойчивость разностной схемы методом гармоник.

3.1. $\dfrac{u_j^{n+1}-u_j^n}{\Delta t}-0.1\dfrac{u_{j+1}^n-u_j^n}{h}=(j-1)h$

3.2. $\dfrac{u_j^{n+1}-u_j^n}{\Delta t}+2\dfrac{u_{j+1}^n-u_j^n}{h}=3n\Delta t$

3.3. $\dfrac{u_j^{n+1}-u_j^n}{\Delta t}+3\dfrac{u_j^n-u_{j-1}^n}{h}=e^{n\Delta t}$

3.4. $\dfrac{u_j^{n+1}-u_j^n}{\Delta t}-4\dfrac{u_j^n-u_{j-1}^n}{h}+2(j-1)h=0$

3.5. $3\dfrac{u_j^{n+1}-u_j^n}{\Delta t}=7\dfrac{u_{j+1}^n-2u_j^n+u_{j-1}^n}{h^2}-2h^2(j-1)^2$

3.6. $\dfrac{u_j^{n+1}-u_j^n}{\Delta t}=5\dfrac{u_{j+1}^n-2u_j^n+u_{j-1}^n}{h^2}-2u_j^n$

3.7. $\dfrac{u_j^{n+1}-u_j^n}{\Delta t}=0.12\dfrac{u_{j+1}^{n+1}-2u_j^{n+1}+u_{j-1}^{n+1}}{h^2}-3u_j^{n+1}+2n\Delta t$

3.8. $\dfrac{u_j^{n+1}-u_j^n}{\Delta t}-6\dfrac{u_{j+1}^{n+1}-u_j^{n+1}}{h}=2n\Delta t(j-1)h$

3.9. $\dfrac{u_j^{n+1}-u_j^n}{\Delta t}+7\dfrac{u_j^{n+1}-u_{j-1}^{n+1}}{h}=13(j-1)h$

3.10. $\dfrac{u_j^{n+1}-u_j^n}{\Delta t}+8\dfrac{u_j^{n+1}-u_{j-1}^{n+1}}{h}=11n\Delta t-3u_j^{n+1}$

3.11. $\dfrac{u_j^{n+1}-u_j^n}{\Delta t}=\dfrac{1}{2}\dfrac{u_{j+1}^n-2u_j^n+u_{j-1}^n}{h^2}+\dfrac{1}{2}\dfrac{u_{j+1}^{n+1}-2u_j^{n+1}+u_{j-1}^{n+1}}{h^2}+(j-1)^2\,h^2$

3.12. $\dfrac{u_j^{n+1}-u_j^n}{\Delta t}+2\dfrac{u_j^n-u_{j-1}^n}{h}=9\dfrac{u_{j+1}^n-2u_j^n+u_{j-1}^n}{h^2}-3n\Delta t$

3.13. $\dfrac{u_j^{n+1}-u_j^n}{\Delta t}+3\dfrac{u_j^{n+1}-u_{j-1}^{n+1}}{h}=8\dfrac{u_{j+1}^{n+1}-2u_j^{n+1}+u_{j-1}^{n+1}}{h^2}-4n\Delta t$

3.14. $\dfrac{u_j^{n+1}-u_j^n}{\Delta t}-5\dfrac{u_{j+1}^n-u_j^n}{h}=6\dfrac{u_{j+1}^n-2u_j^n+u_{j-1}^n}{h^2}+2(j-1)^2\,h^2$

3.15. $\dfrac{u_j^{n+1}-u_j^n}{\Delta t}-9\dfrac{u_{j+1}^{n+1}-u_j^{n+1}}{h}=13\dfrac{u_{j+1}^{n+1}-2u_j^{n+1}+u_{j-1}^{n+1}}{h^2}-u_j^{n+1}-7(j-1)^2\,h^2$

3.16. $\dfrac{u_j^{n+1}-u_j^n}{\Delta t}+7\dfrac{u_{j+1}^n-u_{j-1}^n}{2h}=n^2\Delta t^2$

3.17. $\dfrac{u_j^{n+1}-u_j^n}{\Delta t}+3\dfrac{u_{j+1}^{n+1}-u_{j-1}^{n+1}}{2h}=4\dfrac{u_{j+1}^{n+1}-2u_j^{n+1}+u_{j-1}^{n+1}}{h^2}+5e^{(j-1)h}$

3.18. $\dfrac{u_{j,k}^{n+1}-u_{j,k}^n}{\Delta t}=9\left(\dfrac{u_{j+1,k}^n-2u_{j,k}^n+u_{j-1,k}^n}{h^2}+\dfrac{u_{j,k+1}^n-2u_{j,k}^n+u_{j,k-1}^n}{h^2}\right)+$

$$+(j-1)(k-1)h^2$$

3.19. $\dfrac{u_{j,k}^{n+1}-u_{j,k}^n}{\Delta t}+6\dfrac{u_{j,k}^n-u_{j-1,k}^n}{h}+\dfrac{u_{j,k}^n-u_{j,k-1}^n}{h}=(j-1)h+(k-1)h$

3.20. $\dfrac{u_{j,k}^{n+1}-u_{j,k}^n}{\Delta t}-3\dfrac{u_{j+1,k}^n-u_{j,k}^n}{h}-2\dfrac{u_{j,k+1}^n-u_{j,k}^n}{h}+2n\Delta t=0$

4. Для уравнения: записать явную разностную схему, записать условие устойчивости разностной схемы (без вывода), привести алгоритм решения.

4.1. $\dfrac{\partial u}{\partial t} = 5t\dfrac{\partial^2 u}{\partial x^2} - 3e^t$ $\quad u = u(t, x)$, $\quad u(t=0, x) = x+1$ $\quad \begin{cases} u(t, x=0) = e^t \\ u(t, x=1) = 2e^t \end{cases}$

4.2. $\dfrac{\partial u}{\partial t} + 2\dfrac{\partial u}{\partial x} = 3\dfrac{\partial^2 u}{\partial x^2} + x^2$ $\quad u = u(t, x)$, $\quad u(t=0, x) = x$ $\quad \begin{cases} u(t, x=0) = 0 \\ u(t, x=1) = t+1 \end{cases}$

4.3. $\dfrac{\partial u}{\partial t} - 6t\dfrac{\partial u}{\partial x} = 8x\dfrac{\partial^2 u}{\partial x^2} - 4tx$ $\quad u = u(t, x)$, $\quad u(t=0, x) = -4x$ $\quad \begin{cases} u(t, x=0) = 0 \\ u(t, x=1) = 3t-4 \end{cases}$

4.4. $\dfrac{\partial u}{\partial t} - 0.25\dfrac{\partial u}{\partial x} = 4x - t$ $\quad u = u(t, x)$ $\quad u(t=0, x) = 0$, $\quad u(t, x=1) = 4t$

4.5. $\dfrac{\partial u}{\partial t} + \dfrac{\partial u}{\partial x} = 2 - x - t$ $\quad u = u(t, x)$ $\quad u(t=0, x) = x$, $\quad u(t, x=1) = t$

4.6. $\dfrac{\partial u}{\partial t} = 6\dfrac{\partial^2 u}{\partial x^2} + 6\dfrac{\partial^2 u}{\partial y^2} + xy$ $\quad u = u(t, x, y)$, $\quad u(t=0, x, y) = 0$ $\quad \begin{cases} u(t, x=0, y) = 0 \\ u(t, x=1, y) = ty \end{cases}$ $\quad \begin{cases} u(t, x, y=0) = 0 \\ u(t, x, y=1) = tx \end{cases}$

4.7. $\dfrac{\partial u}{\partial t} = 1.3\left(\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2}\right) + 1$ $\quad u = u(t, x, y)$, $\quad u(t=0, x, y) = xy$ $\quad \begin{cases} u(t, x=0, y) = t \\ u(t, x=1, y) = t + y \end{cases}$ $\quad \begin{cases} u(t, x, y=0) = t \\ u(t, x, y=1) = t + x \end{cases}$

4.8. $3\dfrac{\partial u}{\partial t} = 5.2\left(\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} + \dfrac{\partial^2 u}{\partial z^2}\right) + 3z$ $\quad u = u(t, x, y, z)$, $\quad u(t=0, x, y, z) = xy$

$\begin{cases} u(t, x=0, y, z) = tz \\ u(t, x=1, y, z) = tz + y \end{cases}$ $\quad \begin{cases} u(t, x, y=0, z) = tz \\ u(t, x, y=1, z) = tz + x \end{cases}$ $\quad \begin{cases} u(t, x, y, z=0) = xy \\ u(t, x, y, z=1) = t + xy \end{cases}$

5. Записать для уравнения неявную разностную схему. Привести её к виду, удобному для использования метода прогонки. Проверить сходимость прогонки. Записать рекуррентное прогоночное соотношение. Найти α_1, β_1. Найти u_N^{n+1}. Привести алгоритм решения.

5.1. $2\dfrac{\partial u}{\partial t} = 3\dfrac{\partial^2 u}{\partial x^2} - 6tx$ $\quad u = u(t, x)$, $u(t=0, x) = 0$ $\quad \begin{cases} \dfrac{\partial u}{\partial x}(t, x=0) = 2u(t, x=0) \\ \dfrac{\partial u}{\partial x}(t, x=1) = 3u(t, x=1) \end{cases}$

5.2. $4\dfrac{\partial u}{\partial t} = 5t\dfrac{\partial^2 u}{\partial x^2} - 3u$ $\quad u = u(t, x)$, $u(t=0, x) = 0$ $\quad \begin{cases} \dfrac{\partial u}{\partial x}(t, x=0) = 3 \\ \dfrac{\partial u}{\partial x}(t, x=1) = t^2 \end{cases}$

5.3. $\dfrac{\partial u}{\partial t} = 8\dfrac{\partial^2 u}{\partial x^2} - 4u^2$ $\quad u = u(t, x)$, $u(t=0, x) = 0$ $\quad \begin{cases} \dfrac{\partial u}{\partial x}(t, x=0) = u(t, x=0) \\ u(t, x=1) = e^t - 1 \end{cases}$

5.4. $\dfrac{\partial u}{\partial t} + 6\dfrac{\partial u}{\partial x} = 7\dfrac{\partial^2 u}{\partial x^2} - t^2 x$ $\quad u = u(t, x)$, $u(t=0, x) = x$ $\quad \begin{cases} u(t, x=0) = t^2 \\ \dfrac{\partial u}{\partial x}(t, x=1) = 2u(t, x=1) - 3 \end{cases}$

5.5. $\dfrac{\partial u}{\partial t} - \dfrac{\partial u}{\partial x} = 0.8x\dfrac{\partial^2 u}{\partial x^2} + t - x$ $\quad u = u(t, x)$, $u(t=0, x) = x$ $\quad \begin{cases} \dfrac{\partial u}{\partial x}(t, x=0) = 1 - t \\ u(t, x=1) = 1 \end{cases}$

5.6. $\dfrac{\partial u}{\partial t} - 3\dfrac{\partial u}{\partial x} = 2\dfrac{\partial^2 u}{\partial x^2} + 20u - 3t$ $\quad u = u(t, x)$, $u(t=0, x) = x + 1$ $\quad \begin{cases} \dfrac{\partial u}{\partial x}(t, x=0) = 0 \\ \dfrac{\partial u}{\partial x}(t, x=1) = 0 \end{cases}$

5.7. $\dfrac{\partial u}{\partial t} + 5\dfrac{\partial u}{\partial x} = 9\dfrac{\partial^2 u}{\partial x^2} - 7t^2u^2$ $\quad u = u(t, x)$, $u(t=0, x) = x^2$ $\quad \begin{cases} u(t, x=0) = 2t \\ \dfrac{\partial u}{\partial x}(t, x=1) = 2t \end{cases}$

6. Записать для уравнения схему Кранка–Николсона. Привести её к виду, удобному для использования метода прогонки. Проверить сходимость прогонки. Записать рекуррентное прогоночное соотношение. Найти α_1, β_1. Найти u_N^{n+1}. Привести алгоритм решения.

6.1. $\dfrac{\partial u}{\partial t} = 0.2\dfrac{\partial^2 u}{\partial x^2} - 0.3\,tx$ $\quad u = u(t, x)$, $\quad u(t=0, x) = 0$ $\quad \begin{cases} \dfrac{\partial u}{\partial x}(t, x=0) = 0.2 \\ \dfrac{\partial u}{\partial x}(t, x=1) = 0.4 \end{cases}$

6.2. $\dfrac{\partial u}{\partial t} + 2\dfrac{\partial u}{\partial x} = 8\dfrac{\partial^2 u}{\partial x^2} - 3t$ $\quad u = u(t, x)$, $\quad u(t=0, x) = 0$ $\quad \begin{cases} \dfrac{\partial u}{\partial x}(t, x=0) = 0 \\ u(t, x=1) = t^2 - t \end{cases}$

6.3. $2\dfrac{\partial u}{\partial t} - 3\dfrac{\partial u}{\partial x} = 4\dfrac{\partial^2 u}{\partial x^2} - 7\,tx$ $\quad u = u(t, x)$, $\quad u(t=0, x) = x$ $\quad \begin{cases} u(t, x=0) = 2t \\ \dfrac{\partial u}{\partial x}(t, x=1) = 3u(t, x=1) \end{cases}$

6.4. $\dfrac{1}{2}\dfrac{\partial u}{\partial t} + 5\dfrac{\partial u}{\partial x} = 6\dfrac{\partial^2 u}{\partial x^2} + x$ $\quad u = u(t, x)$, $\quad u(t=0, x) = 1$ $\quad \begin{cases} \dfrac{\partial u}{\partial x}(t, x=0) = 2u(t, x=0) \\ \dfrac{\partial u}{\partial x}(t, x=1) = 0 \end{cases}$

6.5. $\dfrac{\partial u}{\partial t} = 0.9\dfrac{\partial^2 u}{\partial x^2} - 4u$ $\quad u = u(t, x)$, $\quad u(t=0, x) = x$ $\quad \begin{cases} \dfrac{\partial u}{\partial x}(t, x=0) = u(t, x=0) + 2 \\ \dfrac{\partial u}{\partial x}(t, x=1) = u(t, x=1) + 2 \end{cases}$

6.6. $\dfrac{\partial u}{\partial t} = 7\dfrac{\partial u}{\partial x} + 5\dfrac{\partial^2 u}{\partial x^2} - 3u$ $\quad u = u(t, x)$, $\quad u(t=0, x) = x$ $\quad \begin{cases} \dfrac{\partial u}{\partial x}(t, x=0) = 2t + 3 \\ \dfrac{\partial u}{\partial x}(t, x=1) = 2t + 3 \end{cases}$

7. Записать для уравнения разностную схему. Привести её к виду, удобному для использования метода прогонки. Проверить сходимость прогонки. Записать рекуррентное прогоночное соотношение. Найти α_1, β_1. Найти u_N. Привести алгоритм решения.

7.1. $3x\dfrac{d^2u}{dx^2} - 2u = xe^x$ $\qquad u = u(x)$ $\qquad \begin{cases} \dfrac{du}{dx}(x=0) = 2(u(x=0) - 0.5) \\ \dfrac{du}{dx}(x=1) = 2(u(x=1) - 2.5) \end{cases}$

7.2. $\dfrac{d^2u}{dx^2} = 2u(2x+1)$ $\qquad u = u(x)$ $\qquad \begin{cases} \dfrac{du}{dx}(x=0) = 0 \\ \dfrac{du}{dx}(x=1) = 2e \end{cases}$

7.3. $\dfrac{d^2u}{dx^2} = u + 2e^x$ $\qquad u = u(x)$ $\qquad \begin{cases} \dfrac{du}{dx}(x=0) = u(x=0) + 1 \\ \dfrac{du}{dx}(x=1) = 2u(x=1) \end{cases}$

7.4. $x\dfrac{d^2u}{dx^2} = x\dfrac{du}{dx} + u$ $\qquad u = u(x)$ $\qquad \begin{cases} \dfrac{du}{dx}(x=0) = 1 \\ \dfrac{du}{dx}(x=1) = 2e \end{cases}$

7.5. $\dfrac{du}{dx} + \dfrac{d^2u}{dx^2} = u + e^x - e^{-x}$ $\qquad u = u(x)$ $\qquad \begin{cases} \dfrac{du}{dx}(x=0) = 0 \\ \dfrac{du}{dx}(x=1) = u(x=1) - \dfrac{2}{e} \end{cases}$

7.6. $\dfrac{du}{dx} - \dfrac{d^2u}{dx^2} = 2(e^x - u)$ $\qquad u = u(x)$ $\qquad \begin{cases} \dfrac{du}{dx}(x=0) = u(x=0) - 2 \\ \dfrac{du}{dx}(x=1) = e - \dfrac{1}{e} \end{cases}$

8. Записать для уравнения схему расщепления. Для каждой из подсхем записать рекуррентное соотношение. Для тех подсхем, которые решаются методом прогонки: привести к виду, удобному для использования метода прогонки; проверить сходимость прогонки; найти α_1, β_1; найти решение на правой границе. Указать порядок аппроксимации схемы.

8.1. $\dfrac{\partial u}{\partial t} = 0.7\left(\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2}\right) + 1$ $\quad u = u(t, x, y)$, $\quad u(t=0, x, y) = xy$

$$\begin{cases} u(t, x=0, y) = t \\ u(t, x=1, y) = t + y \end{cases} \qquad \begin{cases} \dfrac{\partial u}{\partial y}(t, x, y=0) = x \\ \dfrac{\partial u}{\partial y}(t, x, y=1) = x \end{cases}$$

8.2. $\dfrac{\partial u}{\partial t} + \dfrac{\partial u}{\partial x} - 2\dfrac{\partial u}{\partial y} = t\dfrac{\partial^2 u}{\partial x^2} + 3\dfrac{\partial^2 u}{\partial y^2} - t$ $\quad u = u(t, x, y)$, $\quad u(t=0, x, y) = -xy$

$$\begin{cases} \dfrac{\partial u}{\partial x}(t, x=0, y) = t \\ u(t, x=1, y) = 2t - y \end{cases} \qquad \begin{cases} \dfrac{\partial u}{\partial y}(t, x, y=0) = t \\ u(t, x, y=1) = 2t - x \end{cases}$$

8.3. $\dfrac{\partial u}{\partial t} = 2\dfrac{\partial^2 u}{\partial x^2} + 0.5\dfrac{\partial u}{\partial y} + 2uy$ $\quad u = u(t, x, y)$, $\quad u(t=0, x, y) = 2x$, $\quad u(t, x, y=1) = t + 2x$

$$\begin{cases} \dfrac{\partial u}{\partial x}(t, x=0, y) = u(t, x=0, y) \\ u(t, x=1, y) = ty + 2 \end{cases}$$

8.4. $\dfrac{\partial u}{\partial t} + 5\dfrac{\partial u}{\partial y} = 6\dfrac{\partial^2 u}{\partial x^2} + 2\dfrac{\partial^2 u}{\partial y^2} - tu^2$ $\quad u = u(t, x, y)$, $\quad u(t=0, x, y) = xy$

$$\begin{cases} u(t, x=0, y) = 0 \\ u(t, x=1, y) = t + y \end{cases} \qquad \begin{cases} \dfrac{\partial u}{\partial y}(t, x, y=0) = tx \\ \dfrac{\partial u}{\partial y}(t, x, y=1) = 2tu(t, x, y=1) \end{cases}$$

8.5. $\dfrac{\partial u}{\partial t}+3\dfrac{\partial u}{\partial x}=2\dfrac{\partial u}{\partial y}+0.8\dfrac{\partial^2 u}{\partial y^2}$ $\quad u=u(t,x,y)$, $u(t=0,x,y)=xy$, $u(t,x=0,y)=0$, $\begin{cases}\dfrac{\partial u}{\partial y}(t,x,y=0)=0\\ \dfrac{\partial u}{\partial y}(t,x,y=1)=0\end{cases}$

8.6. $\dfrac{\partial u}{\partial t}-\dfrac{\partial u}{\partial x}=y\dfrac{\partial^2 u}{\partial x^2}+x\dfrac{\partial^2 u}{\partial y^2}-tu$ $\quad u=u(t,x,y)$, $u(t=0,x,y)=0$, $\begin{cases}u(t,x,y=0)=0\\ u(t,x,y=1)=tx\end{cases}$

$$\begin{cases}\dfrac{\partial u}{\partial x}(t,x=0,y)=u(t,x=0,y)-2t\\ \dfrac{\partial u}{\partial x}(t,x=1,y)=u(t,x=1,y)-2t\end{cases}$$

8.7. $\dfrac{\partial u}{\partial t}=6\dfrac{\partial u}{\partial x}+3\dfrac{\partial^2 u}{\partial x^2}-7\dfrac{\partial u}{\partial y}-tu^3$ $\quad u=u(t,x,y)$, $u(t=0,x,y)=0$, $u(t,x,y=0)=0$

$$\begin{cases}u(t,x=0,y)=0\\ \dfrac{\partial u}{\partial x}(t,x=1,y)=3u(t,x=1,y)-5t\end{cases}$$

8.8. $\dfrac{\partial u}{\partial t}+6\dfrac{\partial u}{\partial y}=2\dfrac{\partial^2 u}{\partial x^2}+3\dfrac{\partial^2 u}{\partial y^2}+5\dfrac{\partial^2 u}{\partial z^2}$ $\quad u=u(t,x,y)$, $u(t=0,x,y)=0$

$$\begin{cases}u(t,x=0,y,z)=0\\ u(t,x=1,y,z)=tyz\end{cases}\quad \begin{cases}u(t,x,y=0,z)=0\\ u(t,x,y=1,z)=txz\end{cases}\quad \begin{cases}u(t,x,y,z=0)=0\\ u(t,x,y,z=1)=txy\end{cases}$$

8.9. $\dfrac{\partial u}{\partial t}-3\dfrac{\partial u}{\partial x}-t\dfrac{\partial u}{\partial z}=7\dfrac{\partial^2 u}{\partial x^2}+8\dfrac{\partial^2 u}{\partial y^2}+9\dfrac{\partial^2 u}{\partial z^2}-13u$ $\quad u=u(t,x,y)$, $u(t=0,x,y)=0$

$$\begin{cases}u(t,x=0,y,z)=0\\ u(t,x=1,y,z)=tyz\end{cases}\quad \begin{cases}u(t,x,y=0,z)=0\\ u(t,x,y=1,z)=txz\end{cases}\quad \begin{cases}u(t,x,y,z=0)=0\\ u(t,x,y,z=1)=txy\end{cases}$$

8.10. $\dfrac{\partial u}{\partial t}+\dfrac{\partial u}{\partial x}-3\dfrac{\partial u}{\partial y}-z\dfrac{\partial u}{\partial z}=8xy\dfrac{\partial^2 u}{\partial z^2}+t^2$ $\quad u=u(t,x,y)$, $u(t=0,x,y)=0$

$$u(t,x=0,y,z)=0\qquad u(t,x,y=1,z)=t^2xz\qquad \begin{cases}u(t,x,y,z=0)=0\\ u(t,x,y,z=1)=t^2xy\end{cases}$$

8.11. $\frac{\partial u}{\partial t}-5\frac{\partial u}{\partial z}=2\frac{\partial^2 u}{\partial x^2}+18\frac{\partial^2 u}{\partial y^2}$ $\qquad u=u(t,x,y)$, $u(t=0,x,y)=0$

$\begin{cases} u(t,x=0,y,z)=0 \\ u(t,x=1,y,z)=tyz \end{cases}$ $\begin{cases} u(t,x,y=0,z)=0 \\ u(t,x,y=1,z)=txz \end{cases}$ $u(t,x,y,z=1)=txy$

8.12. $\frac{\partial u}{\partial t}-t\frac{\partial u}{\partial x}-\frac{\partial u}{\partial y}+y\frac{\partial u}{\partial z}=z\frac{\partial^2 u}{\partial x^2}+x\frac{\partial^2 u}{\partial z^2}$ $\qquad u=u(t,x,y)$, $u(t=0,x,y)=0$

$\begin{cases} u(t,x=0,y,z)=0 \\ u(t,x=1,y,z)=tyz \end{cases}$ $u(t,x,y=1,z)=txz$ $\begin{cases} u(t,x,y,z=0)=0 \\ u(t,x,y,z=1)=txy \end{cases}$

8.13. $\frac{\partial u}{\partial t}-4\frac{\partial u}{\partial x}+11\frac{\partial u}{\partial y}=9\frac{\partial^2 u}{\partial y^2}+2\frac{\partial^2 u}{\partial z^2}-3tx$ $\qquad u=u(t,x,y)$, $u(t=0,x,y)=0$

$u(t,x=1,y,z)=tyz$ $\begin{cases} u(t,x,y=0,z)=0 \\ u(t,x,y=1,z)=txz \end{cases}$ $\begin{cases} u(t,x,y,z=0)=0 \\ u(t,x,y,z=1)=txy \end{cases}$

8.14. $\frac{\partial u}{\partial t}+\frac{\partial u}{\partial x}-2\frac{\partial u}{\partial y}+9\frac{\partial u}{\partial z}=5\frac{\partial^2 u}{\partial x^2}+7\frac{\partial^2 u}{\partial y^2}$ $\qquad u=u(t,x,y)$, $u(t=0,x,y)=0$

$\begin{cases} u(t,x=0,y,z)=0 \\ u(t,x=1,y,z)=tyz \end{cases}$ $\begin{cases} u(t,x,y=0,z)=0 \\ u(t,x,y=1,z)=txz \end{cases}$ $u(t,x,y,z=0)=0$

8.15. $\frac{\partial u}{\partial t}-2\frac{\partial u}{\partial x}+4\frac{\partial u}{\partial z}=6\frac{\partial^2 u}{\partial y^2}-9t$ $\qquad u=u(t,x,y)$, $u(t=0,x,y)=0$

$u(t,x=1,y,z)=tyz$ $\begin{cases} u(t,x,y=0,z)=0 \\ u(t,x,y=1,z)=txz \end{cases}$ $u(t,x,y,z=0)=0$

8.16. $\frac{\partial u}{\partial t}+2\frac{\partial u}{\partial x}+3\frac{\partial u}{\partial y}=6\frac{\partial^2 u}{\partial x^2}+5\frac{\partial^2 u}{\partial z^2}-4uy$ $\qquad u=u(t,x,y)$, $u(t=0,x,y)=0$

$\begin{cases} u(t,x=0,y,z)=0 \\ u(t,x=1,y,z)=tyz \end{cases}$ $u(t,x,y=0,z)=0$ $\begin{cases} u(t,x,y,z=0)=0 \\ u(t,x,y,z=1)=txy \end{cases}$

9. Записать для уравнения схему переменных направлений. Для каждой из подсхем записать рекуррентное соотношение. Для тех подсхем, которые решаются методом прогонки: привести к виду, удобному для использования метода прогонки; проверить сходимость прогонки; найти α_1, β_1; найти решение на правой границе. Указать порядок аппроксимации схемы.

9.1. $$\frac{\partial u}{\partial t} = 0.3\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + 1 \qquad \begin{array}{l} u = u(t, x, y) \\ u(t=0, x, y) = xy \end{array} \qquad \begin{cases} u(t, x=0, y) = t \\ u(t, x=1, y) = t + y \end{cases}$$
$$\begin{cases} \dfrac{\partial u}{\partial y}(t, x, y=0) = x \\ \dfrac{\partial u}{\partial y}(t, x, y=1) = x \end{cases}$$

9.2. $$\frac{\partial u}{\partial t} - t\frac{\partial u}{\partial x} + 2t\frac{\partial u}{\partial y} = 4\frac{\partial^2 u}{\partial x^2} + 5\frac{\partial^2 u}{\partial y^2} + t \qquad \begin{array}{l} u = u(t, x, y) \\ u(t=0, x, y) = 0 \end{array}$$
$$\begin{cases} \dfrac{\partial u}{\partial x}(t, x=0, y) = 2t \\ u(t, x=1, y) = 2t(1+y) \end{cases} \qquad \begin{cases} \dfrac{\partial u}{\partial y}(t, x, y=0) = 2t \\ u(t, x, y=1) = 2t(1+x) \end{cases}$$

9.3. $$\frac{\partial u}{\partial t} - 5\frac{\partial u}{\partial y} = 6\frac{\partial^2 u}{\partial x^2} + 7\frac{\partial^2 u}{\partial y^2} - tu^2 \qquad \begin{array}{l} u = u(t, x, y) \\ u(t=0, x, y) = 0 \end{array} \qquad \begin{cases} u(t, x=0, y) = 0 \\ u(t, x=1, y) = ty \end{cases}$$
$$\begin{cases} \dfrac{\partial u}{\partial y}(t, x, y=0) = 2tu(t, x, y=0) \\ \dfrac{\partial u}{\partial y}(t, x, y=1) = tx \end{cases}$$

9.4. $$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} + 3\frac{\partial u}{\partial y} = 3\frac{\partial^2 u}{\partial x^2} + 6\frac{\partial^2 u}{\partial y^2} - 9t^2 \qquad \begin{array}{l} u = u(t, x, y) \\ u(t=0, x, y) = 0 \end{array}$$
$$\begin{cases} \dfrac{\partial u}{\partial x}(t, x=0, y) = t \\ \dfrac{\partial u}{\partial x}(t, x=1, y) = 1 + t \end{cases} \qquad \begin{cases} \dfrac{\partial u}{\partial y}(t, x, y=0) = t \\ \dfrac{\partial u}{\partial y}(t, x, y=1) = 1 + t \end{cases}$$

9.5. $\dfrac{\partial u}{\partial t}+t\dfrac{\partial u}{\partial x}=y\dfrac{\partial^2 u}{\partial x^2}+x\dfrac{\partial^2 u}{\partial y^2}-tu$ $\quad u=u(t,x,y)$ $\quad u(t=0,x,y)=0$ $\quad \begin{cases} u(t,x,y=0)=0 \\ u(t,x,y=1)=tx \end{cases}$

$$\begin{cases} u(t,x=0,y)=0 \\ \dfrac{\partial u}{\partial x}(t,x=1,y)=5u(t,x=1,y)-3t \end{cases}$$

9.6. $\dfrac{\partial u}{\partial t}=6\dfrac{\partial u}{\partial x}+7\dfrac{\partial u}{\partial y}+3\dfrac{\partial^2 u}{\partial x^2}-tu^3$ $\quad u=u(t,x,y)$ $\quad u(t=0,x,y)=0$ $\quad u(t,x,y=1)=tx$

$$\begin{cases} \dfrac{\partial u}{\partial x}(t,x=0,y)=u(t,x=0,y)+2t \\ \dfrac{\partial u}{\partial x}(t,x=1,y)=u(t,x=1,y)+2t \end{cases}$$

9.7. $\dfrac{\partial u}{\partial t}=2\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial u}{\partial y}+2uy$ $\quad u=u(t,x,y)$ $\quad u(t=0,x,y)=2x$ $\quad u(t,x,y=1)=t+2x$

$$\begin{cases} \dfrac{\partial u}{\partial x}(t,x=0,y)=u(t,x=0,y) \\ \dfrac{\partial u}{\partial x}(t,x=1,y)=t \end{cases}$$

9.8. $\dfrac{\partial u}{\partial t}=5t\dfrac{\partial u}{\partial x}+6t\dfrac{\partial u}{\partial y}+7\dfrac{\partial^2 u}{\partial x^2}+8\dfrac{\partial^2 u}{\partial y^2}-12tu$ $\quad u=u(t,x,y)$ $\quad u(t=0,x,y)=1$

$$\begin{cases} u(t,x=0,y)=1 \\ u(t,x=1,y)=ty+1 \end{cases} \qquad \begin{cases} \dfrac{\partial u}{\partial y}(t,x,y=0)=u(t,x,y=0)+5 \\ \dfrac{\partial u}{\partial y}(t,x,y=1)=2u(t,x,y=1)-7 \end{cases}$$

9.9. $\dfrac{\partial u}{\partial t}=y\dfrac{\partial u}{\partial x}-2t\dfrac{\partial u}{\partial y}+8\dfrac{\partial^2 u}{\partial y^2}-3x$ $\quad u=u(t,x,y)$ $\quad u(t=0,x,y)=0$ $\quad u(t,x=1,y)=ty$

$$\begin{cases} u(t,x,y=0)=0 \\ \dfrac{\partial u}{\partial y}(t,x,y=1)=xu(t,x,y=1)-t^2 \end{cases}$$

9.10. $\dfrac{\partial u}{\partial t}+3\dfrac{\partial u}{\partial x}=-2\dfrac{\partial u}{\partial y}+5\dfrac{\partial^2 u}{\partial y^2}$ $\quad u=u(t,x,y)$ $\quad u(t=0,x,y)=y^2$ $\quad u(t,x=0,y)=y^2$ $\quad \begin{cases} \dfrac{\partial u}{\partial y}(t,x,y=0)=0 \\ \dfrac{\partial u}{\partial y}(t,x,y=1)=1 \end{cases}$

9.11. $\dfrac{\partial u}{\partial t} = 2\dfrac{\partial u}{\partial x} + 3\dfrac{\partial^2 u}{\partial x^2} + 18\dfrac{\partial^2 u}{\partial y^2} + txy$ $\qquad u = u(t, x, y)$

$u(t=0, x, y) = 0$

$\begin{cases} u(t, x=0, y) = t \\ \dfrac{\partial u}{\partial x}(t, x=1, y) = 0 \end{cases}$ $\qquad \begin{cases} \dfrac{\partial u}{\partial y}(t, x, y=0) = u(t, x, y=0) \\ \dfrac{\partial u}{\partial y}(t, x, y=1) = 0 \end{cases}$

9.12. $\dfrac{\partial u}{\partial t} + t\dfrac{\partial u}{\partial x} = 2\dfrac{\partial^2 u}{\partial x^2} - 3\dfrac{\partial u}{\partial y} - 2u^2$ $\qquad u = u(t, x, y)$ $\qquad u(t=0, x, y) = x + y^2$

$u(t, x, y=0) = t + x$

$\begin{cases} \dfrac{\partial u}{\partial x}(t, x=0, y) = 2y \\ \dfrac{\partial u}{\partial x}(t, x=1, y) = 2y\,u(t, x=1, y) \end{cases}$

9.13. $\dfrac{\partial u}{\partial t} = 3\dfrac{\partial u}{\partial x} + 4\dfrac{\partial u}{\partial y} + t\dfrac{\partial^2 u}{\partial y^2} + txy$ $\qquad u = u(t, x, y)$ $\qquad u(t=0, x, y) = 0$

$u(t, x=1, y) = t\,y$

$\begin{cases} \dfrac{\partial u}{\partial y}(t, x, y=0) = 0 \\ \dfrac{\partial u}{\partial y}(t, x, y=1) = 3u(t, x, y=1) \end{cases}$

9.14. $\dfrac{\partial u}{\partial t} = 2\dfrac{\partial^2 u}{\partial x^2} + 3\dfrac{\partial^2 u}{\partial y^2} - 0.8t\dfrac{\partial u}{\partial y}$ $\qquad u = u(t, x, y)$

$u(t=0, x, y) = 0$ $\qquad \begin{cases} u(t, x, y=0) = t \\ \dfrac{\partial u}{\partial y}(t, x, y=1) = 2tx \end{cases}$

$\begin{cases} \dfrac{\partial u}{\partial x}(t, x=0, y) = 6u(t, x=0, y) \\ \dfrac{\partial u}{\partial x}(t, x=1, y) = 0 \end{cases}$

9.15. $\dfrac{\partial u}{\partial t} + 5\dfrac{\partial u}{\partial x} = 9\dfrac{\partial u}{\partial y} + 4\dfrac{\partial^2 u}{\partial x^2} + 7t\dfrac{\partial^2 u}{\partial y^2} - 3u$ $\qquad u = u(t, x, y)$

$u(t=0, x, y) = 3xy$

$\begin{cases} u(t, x=0, y) = t \\ \dfrac{\partial u}{\partial x}(t, x=1, y) = 2u(t, x=1, y) - 3 \end{cases}$ $\qquad \begin{cases} \dfrac{\partial u}{\partial y}(t, x, y=0) = 0 \\ u(t, x, y=1) = t + 3x \end{cases}$

10. Записать для уравнения схему предиктор-корректор. Для каждой из подсхем записать рекуррентное соотношение. Указать порядок аппроксимации схемы.

10.1. $\dfrac{\partial u}{\partial t} + t\dfrac{\partial u}{\partial x} = y\dfrac{\partial^2 u}{\partial x^2} + x\dfrac{\partial^2 u}{\partial y^2} - tu^3$

$$u = u(t, x, y)$$
$$u(t=0, x, y) = 0$$

$$\begin{cases} \dfrac{\partial u}{\partial x}(t, x=0, y) = 2u(t, x=0, y) + 3 \\ \dfrac{\partial u}{\partial x}(t, x=1, y) = 2u(t, x=1, y) + 3 \end{cases}$$

$$\begin{cases} u(t, x, y=0) = 0 \\ u(t, x, y=1) = tx \end{cases}$$

10.2. $\dfrac{\partial u}{\partial t} = 2\dfrac{\partial^2 u}{\partial x^2} + 3\dfrac{\partial^2 u}{\partial y^2} + 8t\dfrac{\partial u}{\partial y} + 5t$

$$u = u(t, x, y)$$
$$u(t=0, x, y) = 0$$

$$\begin{cases} \dfrac{\partial u}{\partial x}(t, x=0, y) = 2u(t, x=0, y) \\ \dfrac{\partial u}{\partial x}(t, x=1, y) = 2u(t, x=1, y) \end{cases}$$

$$\begin{cases} \dfrac{\partial u}{\partial y}(t, x, y=0) = t \\ \dfrac{\partial u}{\partial y}(t, x, y=1) = t \end{cases}$$

10.3. $\dfrac{\partial u}{\partial t} - t\dfrac{\partial u}{\partial x} = 4\dfrac{\partial^2 u}{\partial y^2} - t\dfrac{\partial u}{\partial y} + 3u^2(xy+1)$

$$u = u(t, x, y)$$
$$u(t=0, x, y) = 0$$

$$u(t, x=1, y) = ty + 1$$

$$\begin{cases} u(t, x, y=0) = 0 \\ u(t, x, y=1) = tx + 1 \end{cases}$$

10.4. $\dfrac{\partial u}{\partial t} - 12\dfrac{\partial u}{\partial x} = 5\dfrac{\partial u}{\partial y} + 7\dfrac{\partial^2 u}{\partial x^2} + 11\dfrac{\partial^2 u}{\partial y^2} - 13ut$

$$u = u(t, x, y)$$
$$u(t=0, x, y) = -3$$

$$\begin{cases} u(t, x=0, y) = -3 \\ u(t, x=1, y) = ty - 3 \end{cases}$$

$$\begin{cases} \dfrac{\partial u}{\partial y}(t, x, y=0) = 0 \\ u(t, x, y=1) = tx - 3 \end{cases}$$

10.5. $\dfrac{\partial u}{\partial t} = 2\dfrac{\partial^2 u}{\partial x^2} - x\dfrac{\partial u}{\partial y} + tuy$

$$u = u(t, x, y)$$
$$u(t=0, x, y) = 0$$
$$u(t, x, y=0) = t$$

$$\begin{cases} u(t, x=0, y) = t \\ \dfrac{\partial u}{\partial x}(t, x=1, y) = 0 \end{cases}$$

10.6. $$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}+0.3\frac{\partial^2 u}{\partial y^2}-0.5\frac{\partial u}{\partial y}+2t$$

$$u=u(t,x,y)$$
$$u(t=0,x,y)=0$$

$$\begin{cases} u(t,x=0,y)=0 \\ \dfrac{\partial u}{\partial x}(t,x=1,y)=0 \end{cases} \qquad \begin{cases} u(t,x,y=0)=0 \\ \dfrac{\partial u}{\partial y}(t,x,y=1)=0 \end{cases}$$

10.7. $$\frac{\partial u}{\partial t}+t\frac{\partial u}{\partial x}=0.6\frac{\partial^2 u}{\partial y^2}+t^3$$

$$u=u(t,x,y)$$
$$u(t=0,x,y)=x+y$$

$$u(t,x=0,y)=t^3+y \qquad \begin{cases} \dfrac{\partial u}{\partial y}(t,x,y=0)=3t^2 \\ \dfrac{\partial u}{\partial y}(t,x,y=1)=3t^2 \end{cases}$$

10.8. $$\frac{\partial u}{\partial t}-2t\frac{\partial u}{\partial x}=8\frac{\partial^2 u}{\partial x^2}+5\frac{\partial^2 u}{\partial y^2}-3tu^2$$

$$u=u(t,x,y)$$
$$u(t=0,x,y)=0$$

$$\begin{cases} \dfrac{\partial u}{\partial x}(t,x=0,y)=0 \\ \dfrac{\partial u}{\partial x}(t,x=1,y)=t \end{cases} \qquad \begin{cases} u(t,x,y=0)=0 \\ u(t,x,y=1)=tx \end{cases}$$

10.9. $$\frac{\partial u}{\partial t}=2\frac{\partial^2 u}{\partial x^2}+0.5\frac{\partial u}{\partial y}+2uy$$

$$u=u(t,x,y)$$
$$u(t=0,x,y)=2x$$

$$\begin{cases} \dfrac{\partial u}{\partial x}(t,x=0,y)=u(t,x=0,y) \\ u(t,x=1,y)=ty+2 \end{cases} \qquad u(t,x,y=1)=t+2x$$

10.10. $$\frac{\partial u}{\partial t}-t\frac{\partial u}{\partial x}+2t\frac{\partial u}{\partial y}=4\frac{\partial^2 u}{\partial x^2}+5\frac{\partial^2 u}{\partial y^2}+t^2xy$$

$$u=u(t,x,y)$$
$$u(t=0,x,y)=0$$

$$\begin{cases} \dfrac{\partial u}{\partial x}(t,x=0,y)=t \\ u(t,x=1,y)=t(y+1) \end{cases} \qquad \begin{cases} \dfrac{\partial u}{\partial y}(t,x,y=0)=t \\ u(t,x,y=1)=t(x+1) \end{cases}$$

10.11. $$\frac{\partial u}{\partial t}+3\frac{\partial u}{\partial x}=2\frac{\partial u}{\partial y}+0.8\frac{\partial^2 u}{\partial y^2}+t$$

$$u=u(t,x,y)$$
$$u(t=0,x,y)=0$$
$$u(t,x=0,y)=0$$

$$\begin{cases} \dfrac{\partial u}{\partial y}(t,x,y=0)=0 \\ \dfrac{\partial u}{\partial y}(t,x,y=1)=0 \end{cases}$$

10.12. $\dfrac{\partial u}{\partial t}+t\dfrac{\partial u}{\partial x}=2\dfrac{\partial^2 u}{\partial x^2}-3\dfrac{\partial u}{\partial y}-2u^2$

$u=u(t,x,y)$

$u(t=0,x,y)=2xy$

$$\begin{cases}\dfrac{\partial u}{\partial x}(t,x=0,y)=2y\\ \dfrac{\partial u}{\partial x}(t,x=1,y)=2y\end{cases}$$

$u(t,x,y=0)=t$

10.13. $9\dfrac{\partial u}{\partial t}+3\dfrac{\partial u}{\partial x}+7\dfrac{\partial u}{\partial y}-11\dfrac{\partial u}{\partial z}=6\dfrac{\partial^2 u}{\partial x^2}+5\dfrac{\partial^2 u}{\partial z^2}-3u$

$u=u(t,x,y,z)$

$u(t=0,x,y,z)=e^{x+y+z}$

$u(t,x,y=0,z)=e^{t+x+z}$

$$\begin{cases}u(t,x=0,y,z)=e^{t+y+z}\\ u(t,x=1,y,z)=e^{t+y+z+1}\end{cases}\qquad \begin{cases}u(t,x,y,z=0)=e^{t+x+y}\\ u(t,x,y,z=1)=e^{t+x+y+1}\end{cases}$$

10.14. $\dfrac{\partial u}{\partial t}-4\dfrac{\partial u}{\partial x}+5\dfrac{\partial u}{\partial y}+6\dfrac{\partial u}{\partial z}=7\dfrac{\partial^2 u}{\partial x^2}+2\dfrac{\partial^2 u}{\partial y^2}-u$

$u=u(t,x,y,z)$

$u(t=0,x,y,z)=e^{x+y+z}$

$$\begin{cases}u(t,x,y=0,z)=e^{t+x+z}\\ u(t,x,y=1,z)=e^{t+x+z+1}\end{cases}$$

$$\begin{cases}u(t,x=0,y,z)=e^{t+y+z}\\ u(t,x=1,y,z)=e^{t+y+z+1}\end{cases}$$

$u(t,x,y,z=0)=e^{t+x+y}$

10.15. $4\dfrac{\partial u}{\partial t}+7\dfrac{\partial u}{\partial y}-3\dfrac{\partial u}{\partial z}=6\dfrac{\partial^2 u}{\partial x^2}-2u$

$u=u(t,x,y,z)$

$u(t=0,x,y,z)=e^{x+y+z}$

$$\begin{cases}u(t,x=0,y,z)=e^{t+y+z}\\ u(t,x=1,y,z)=e^{t+y+z+1}\end{cases}$$

$u(t,x,y=0,z)=e^{t+x+z}$

$u(t,x,y,z=1)=e^{t+x+y+1}$

10.16. $\dfrac{\partial u}{\partial t}+9\dfrac{\partial u}{\partial y}=7\dfrac{\partial^2 u}{\partial x^2}+4\dfrac{\partial^2 u}{\partial z^2}-u$

$u=u(t,x,y,z)$

$u(t=0,x,y,z)=e^{x+y+z}$

$u(t,x,y=0,z)=e^{t+x+z}$

$$\begin{cases}u(t,x=0,y,z)=e^{t+y+z}\\ u(t,x=1,y,z)=e^{t+y+z+1}\end{cases}\qquad \begin{cases}u(t,x,y,z=0)=e^{t+x+y}\\ u(t,x,y,z=1)=e^{t+x+y+1}\end{cases}$$

10.17. $2\frac{\partial u}{\partial t}-3\frac{\partial u}{\partial x}+4\frac{\partial u}{\partial z}=\frac{\partial^2 u}{\partial x^2}+5\frac{\partial^2 u}{\partial y^2}-3u$

$u=u(t,x,y,z)$

$u(t=0,x,y,z)=e^{x+y+z}$

$\begin{cases} u(t,x=0,y,z)=e^{t+y+z} \\ u(t,x=1,y,z)=e^{t+y+z+1} \end{cases}$

$\begin{cases} u(t,x,y=0,z)=e^{t+x+z} \\ u(t,x,y=1,z)=e^{t+x+z+1} \end{cases}$

$u(t,x,y,z=0)=e^{t+x+y}$

10.18. $3\frac{\partial u}{\partial t}+2\frac{\partial u}{\partial x}-\frac{\partial u}{\partial y}+5\frac{\partial u}{\partial z}=6\frac{\partial^2 u}{\partial y^2}+2\frac{\partial^2 u}{\partial z^2}+u$

$u=u(t,x,y,z)$

$u(t=0,x,y,z)=e^{x+y+z}$

$u(t,x=0,y,z)=e^{t+y+z}$

$\begin{cases} u(t,x,y=0,z)=e^{t+x+z} \\ u(t,x,y=1,z)=e^{t+x+z+1} \end{cases}$

$\begin{cases} u(t,x,y,z=0)=e^{t+x+y} \\ u(t,x,y,z=1)=e^{t+x+y+1} \end{cases}$

10.19. $2\frac{\partial u}{\partial t}+3\frac{\partial u}{\partial y}-4\frac{\partial u}{\partial z}=5\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial z^2}-5u$

$u=u(t,x,y,z)$

$u(t=0,x,y,z)=e^{x+y+z}$

$\begin{cases} u(t,x=0,y,z)=e^{t+y+z} \\ u(t,x=1,y,z)=e^{t+y+z+1} \end{cases}$

$u(t,x,y=0,z)=e^{t+x+z}$

$\begin{cases} u(t,x,y,z=0)=e^{t+x+y} \\ u(t,x,y,z=1)=e^{t+x+y+1} \end{cases}$

10.20. $\frac{\partial u}{\partial t}-3\frac{\partial u}{\partial x}+9\frac{\partial u}{\partial z}=8\frac{\partial^2 u}{\partial y^2}-u$

$\begin{cases} u(t,x,y=0,z)=e^{t+x+z} \\ u(t,x,y=1,z)=e^{t+x+z+1} \end{cases}$

$u=u(t,x,y,z)$

$u(t=0,x,y,z)=e^{x+y+z}$

$u(t,x=1,y,z)=e^{t+y+z+1}$

$u(t,x,y,z=0)=e^{t+x+y}$

10.21. $2\frac{\partial u}{\partial t}+5\frac{\partial u}{\partial x}-4\frac{\partial u}{\partial y}=6\frac{\partial^2 u}{\partial z^2}-3u$

$u(t,x=0,y,z)=e^{t+y+z}$

$u(t,x,y=1,z)=e^{t+x+z+1}$

$u=u(t,x,y,z)$

$u(t=0,x,y,z)=e^{x+y+z}$

$\begin{cases} u(t,x,y,z=0)=e^{t+x+y} \\ u(t,x,y,z=1)=e^{t+x+y+1} \end{cases}$

10.22. $2\dfrac{\partial u}{\partial t}-\dfrac{\partial u}{\partial z}=3\dfrac{\partial^2 u}{\partial x^2}+5\dfrac{\partial^2 u}{\partial y^2}-7u$

$u=u(t,x,y,z)$

$u(t=0,x,y,z)=e^{x+y+z}$

$\begin{cases} u(t,x=0,y,z)=e^{t+y+z} \\ u(t,x=1,y,z)=e^{t+y+z+1} \end{cases}$

$\begin{cases} u(t,x,y=0,z)=e^{t+x+z} \\ u(t,x,y=1,z)=e^{t+x+z+1} \end{cases}$

$u(t,x,y,z=1)=e^{t+x+y+1}$

10.23. $8\dfrac{\partial u}{\partial t}+\dfrac{\partial u}{\partial x}-2\dfrac{\partial u}{\partial y}=5\dfrac{\partial^2 u}{\partial x^2}+3\dfrac{\partial^2 u}{\partial z^2}-u$

$u=u(t,x,y,z)$

$u(t=0,x,y,z)=e^{x+y+z}$

$\begin{cases} u(t,x=0,y,z)=e^{t+y+z} \\ u(t,x=1,y,z)=e^{t+y+z+1} \end{cases}$

$u(t,x,y=1,z)=e^{t+x+z+1}$

$\begin{cases} u(t,x,y,z=0)=e^{t+x+y} \\ u(t,x,y,z=1)=e^{t+x+y+1} \end{cases}$

10.24. $3\dfrac{\partial u}{\partial t}+2\dfrac{\partial u}{\partial x}=4\dfrac{\partial^2 u}{\partial y^2}+9\dfrac{\partial^2 u}{\partial z^2}-8u$

$u=u(t,x,y,z)$

$u(t=0,x,y,z)=e^{x+y+z}$

$u(t,x=0,y,z)=e^{t+y+z}$

$\begin{cases} u(t,x,y=0,z)=e^{t+x+z} \\ u(t,x,y=1,z)=e^{t+x+z+1} \end{cases}$

$\begin{cases} u(t,x,y,z=0)=e^{t+x+y} \\ u(t,x,y,z=1)=e^{t+x+y+1} \end{cases}$

10.25. $3\dfrac{\partial u}{\partial t}-2\dfrac{\partial u}{\partial x}-\dfrac{\partial u}{\partial y}+6\dfrac{\partial u}{\partial z}=2\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}+5\dfrac{\partial^2 u}{\partial z^2}-2u$

$u=u(t,x,y,z)$

$u(t=0,x,y,z)=e^{x+y+z}$

$\begin{cases} u(t,x=0,y,z)=e^{t+y+z} \\ u(t,x=1,y,z)=e^{t+y+z+1} \end{cases}$

$\begin{cases} u(t,x,y=0,z)=e^{t+x+z} \\ u(t,x,y=1,z)=e^{t+x+z+1} \end{cases}$

$\begin{cases} u(t,x,y,z=0)=e^{t+x+y} \\ u(t,x,y,z=1)=e^{t+x+y+1} \end{cases}$

10.26. $\dfrac{\partial u}{\partial t}+5\dfrac{\partial u}{\partial x}-2\dfrac{\partial u}{\partial z}=5\dfrac{\partial^2 u}{\partial y^2}+6\dfrac{\partial^2 u}{\partial z^2}-7u$

$u=u(t,x,y,z)$

$u(t=0,x,y,z)=e^{x+y+z}$

$u(t,x=0,y,z)=e^{t+y+z}$

$\begin{cases} u(t,x,y=0,z)=e^{t+x+z} \\ u(t,x,y=1,z)=e^{t+x+z+1} \end{cases}$

$\begin{cases} u(t,x,y,z=0)=e^{t+x+y} \\ u(t,x,y,z=1)=e^{t+x+y+1} \end{cases}$

11. Выбрать соответствующие уравнению граничные условия. Записать явную разностную схему. Записать условие устойчивости на шаг. Записать рекуррентное соотношение.

11.1. $\dfrac{\partial u}{\partial t}+0.5\dfrac{\partial u}{\partial x}=-2\dfrac{\partial u}{\partial y}+xy$ $\quad u=u(t,x,y)$, $\quad u(t=0,x,y)=0$

$$\begin{cases} u(t,x=0,y)=0 \\ u(t,x=1,y)=ty \end{cases} \qquad \begin{cases} u(t,x,y=0)=0 \\ u(t,x,y=1)=tx \end{cases}$$

11.2. $\dfrac{\partial u}{\partial t}-3\dfrac{\partial u}{\partial x}=4\dfrac{\partial u}{\partial y}+5t$ $\quad u=u(t,x,y)$, $\quad u(t=0,x,y)=xy$

$$\begin{cases} u(t,x=0,y)=t \\ u(t,x=1,y)=t+y \end{cases} \qquad \begin{cases} u(t,x,y=0)=t \\ u(t,x,y=1)=t+x \end{cases}$$

11.3. $\dfrac{\partial u}{\partial t}+0.2\dfrac{\partial u}{\partial x}=0.2\dfrac{\partial u}{\partial y}-0.4(x+y)$ $\quad u=u(t,x,y)$, $\quad u(t=0,x,y)=1$

$$\begin{cases} u(t,x=0,y)=1 \\ u(t,x=1,y)=1+ty \end{cases} \qquad \begin{cases} u(t,x,y=0)=1 \\ u(t,x,y=1)=1+tx \end{cases}$$

11.4. $\dfrac{\partial u}{\partial t}=6\dfrac{\partial u}{\partial x}-5\dfrac{\partial u}{\partial y}+3txy$ $\quad u=u(t,x,y)$, $\quad u(t=0,x,y)=0$

$$\begin{cases} u(t,x=0,y)=0 \\ u(t,x=1,y)=t \end{cases} \qquad \begin{cases} u(t,x,y=0)=0 \\ u(t,x,y=1)=t \end{cases}$$

11.5. $\dfrac{\partial u}{\partial t}+3\dfrac{\partial u}{\partial x}=4\dfrac{\partial u}{\partial y}-\dfrac{\partial u}{\partial z}-txyz$ $\quad u=u(t,x,y,z)$, $\quad u(t=0,x,y,z)=0$

$$\begin{cases} u(t,x=0,y,z)=0 \\ u(t,x=1,y,z)=tyz \end{cases} \qquad \begin{cases} u(t,x,y=0,z)=0 \\ u(t,x,y=1,z)=txz \end{cases} \qquad \begin{cases} u(t,x,y,z=0)=0 \\ u(t,x,y,z=1)=txy \end{cases}$$

11.6. $\dfrac{\partial u}{\partial t}-2\dfrac{\partial u}{\partial z}=7\dfrac{\partial u}{\partial x}-5\dfrac{\partial u}{\partial y}$ $\quad u=u(t,x,y,z)$, $\quad u(t=0,x,y,z)=0$

$$\begin{cases} u(t,x=0,y,z)=0 \\ u(t,x=1,y,z)=tyz \end{cases} \qquad \begin{cases} u(t,x,y=0,z)=0 \\ u(t,x,y=1,z)=txz \end{cases} \qquad \begin{cases} u(t,x,y,z=0)=0 \\ u(t,x,y,z=1)=txy \end{cases}$$

12. Выбрать соответствующие уравнению граничные условия. Записать схему расщепления. Для каждой из подсхем записать рекуррентное соотношение.

12.1. $\frac{\partial u}{\partial t} - 2x\frac{\partial u}{\partial y} = y\frac{\partial u}{\partial x} + t^2$

$u = u(t, x, y)$

$u(t = 0, x, y) = xy$

$\begin{cases} u(t, x = 0, y) = t^2 \\ u(t, x = 1, y) = t^2 + y \end{cases}$ $\begin{cases} u(t, x, y = 0) = t^2 \\ u(t, x, y = 1) = t^2 + x \end{cases}$

12.2. $\frac{\partial u}{\partial t} + 3t\frac{\partial u}{\partial x} = 8\frac{\partial u}{\partial y} = -2u$

$u = u(t, x, y)$

$u(t = 0, x, y) = 0$

$\begin{cases} u(t, x = 0, y) = 0 \\ u(t, x = 1, y) = 2ty \end{cases}$ $\begin{cases} u(t, x, y = 0) = 0 \\ u(t, x, y = 1) = 2tx \end{cases}$

12.3. $2\frac{\partial u}{\partial t} = 0.3\frac{\partial u}{\partial x} - 0.2\frac{\partial u}{\partial y}$

$u = u(t, x, y)$

$u(t = 0, x, y) = 0$

$\begin{cases} u(t, x = 0, y) = 0 \\ u(t, x = 1, y) = t \end{cases}$ $\begin{cases} u(t, x, y = 0) = 0 \\ u(t, x, y = 1) = t \end{cases}$

12.4. $\frac{\partial u}{\partial t} + 7t\frac{\partial u}{\partial y} = txy - 6\frac{\partial u}{\partial x}$

$u = u(t, x, y)$

$u(t = 0, x, y) = 0$

$\begin{cases} u(t, x = 0, y) = 0 \\ u(t, x = 1, y) = ty \end{cases}$ $\begin{cases} u(t, x, y = 0) = 0 \\ u(t, x, y = 1) = tx \end{cases}$

12.5. $\frac{\partial u}{\partial t} = 2.5\frac{\partial u}{\partial x} - 3t\frac{\partial u}{\partial y} + 5.2\frac{\partial u}{\partial z} - 1.8tu$

$u = u(t, x, y, z)$

$u(t = 0, x, y, z) = 0$

$\begin{cases} u(t, x = 0, y, z) = 0 \\ u(t, x = 1, y, z) = tyz \end{cases}$ $\begin{cases} u(t, x, y = 0, z) = 0 \\ u(t, x, y = 1, z) = txz \end{cases}$ $\begin{cases} u(t, x, y, z = 0) = 0 \\ u(t, x, y, z = 1) = txy \end{cases}$

12.6. $\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} + 2\frac{\partial u}{\partial z} = \frac{\partial u}{\partial x} - u$

$u = u(t, x, y, z)$

$u(t = 0, x, y, z) = 0$

$\begin{cases} u(t, x = 0, y, z) = 0 \\ u(t, x = 1, y, z) = tyz \end{cases}$ $\begin{cases} u(t, x, y = 0, z) = 0 \\ u(t, x, y = 1, z) = txz \end{cases}$ $\begin{cases} u(t, x, y, z = 0) = 0 \\ u(t, x, y, z = 1) = txy \end{cases}$

13. Выбрать соответствующие уравнению граничные условия. Записать схему переменных направлений. Для каждой из подсхем записать рекуррентное соотношение.

13.1. $\frac{\partial u}{\partial t} + 5\frac{\partial u}{\partial y} = 6\frac{\partial u}{\partial x} + t^2$

$$u = u(t, x, y)$$
$$u(t=0, x, y) = 0$$
$$\begin{cases} u(t, x=0, y) = 0 \\ u(t, x=1, y) = t^2 y \end{cases} \qquad \begin{cases} u(t, x, y=0) = 0 \\ u(t, x, y=1) = t^2 x \end{cases}$$

13.2. $\frac{\partial u}{\partial t} = 2t\frac{\partial u}{\partial y} - 3t\frac{\partial u}{\partial x} + txy$

$$u = u(t, x, y)$$
$$u(t=0, x, y) = 0$$
$$\begin{cases} u(t, x=0, y) = 0 \\ u(t, x=1, y) = ty \end{cases} \qquad \begin{cases} u(t, x, y=0) = 0 \\ u(t, x, y=1) = tx \end{cases}$$

13.3. $\frac{1}{2}\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x} + 6t\frac{\partial u}{\partial y} - 4tu$

$$u = u(t, x, y)$$
$$u(t=0, x, y) = 1$$
$$\begin{cases} u(t, x=0, y) = 1 \\ u(t, x=1, y) = 1+t \end{cases} \qquad \begin{cases} u(t, x, y=0) = 1 \\ u(t, x, y=1) = 1+t \end{cases}$$

13.4. $\frac{\partial u}{\partial t} + 8t\frac{\partial u}{\partial x} = 9t(x+y) - 7x\frac{\partial u}{\partial y}$

$$u = u(t, x, y)$$
$$u(t=0, x, y) = xy$$
$$\begin{cases} u(t, x=0, y) = t \\ u(t, x=1, y) = t + y \end{cases} \qquad \begin{cases} u(t, x, y=0) = t \\ u(t, x, y=1) = t + x \end{cases}$$

14. Методом Рунге-Кутта 2-го порядка найти значение $u(t^0+\tau)$.

14.1.	$\frac{du}{dt}=e^t\cdot u$	$u=u(t)$	$u(t=0)=e$	$\tau=0{,}1$
14.2.	$\frac{du}{dt}=t\cdot u$	$u=u(t)$	$u(t=0)=1$	$\tau=0{,}1$
14.3.	$2\frac{du}{dt}=e^t+u$	$u=u(t)$	$u(t=0)=1$	$\tau=0{,}1$
14.4.	$\frac{du}{dt}=3t^2e^t+u$	$u=u(t)$	$u(t=0)=0$	$\tau=1$
14.5.	$\frac{du}{dt}=\frac{u}{2t}$	$u=u(t)$	$u(t=1)=1$	$\tau=0{,}1$
14.6.	$\frac{du}{dt}=\frac{2u}{t+3}$	$u=u(t)$	$u(t=0)=9$	$\tau=1$
14.7.	$\frac{du}{dt}=ut^2$	$u=u(t)$	$u(t=0)=1$	$\tau=1$
14.8.	$\frac{du}{dt}=\frac{3u}{t-1}$	$u=u(t)$	$u(t=2)=1$	$\tau=0{,}1$
14.9.	$\frac{du}{dt}=\frac{3u}{t-1}$	$u=u(t)$	$u(t=3)=8$	$\tau=0{,}2$
14.10.	$\frac{du}{dt}=\frac{t}{u}$	$u=u(t)$	$u(t=1)=\sqrt{2}$	$\tau=0{,}1$
14.11.	$\frac{du}{dt}=\frac{12tu}{2t^3+5}$	$u=u(t)$	$u(t=0)=25$	$\tau=0{,}5$
14.12.	$\frac{du}{dt}=\frac{2u}{t-4}$	$u=u(t)$	$u(t=0)=4$	$\tau=0{,}2$
14.13.	$\frac{du}{dt}=\frac{2u+2ue^t}{t+e^t}$	$u=u(t)$	$u(t=0)=1$	$\tau=0{,}1$
14.14.	$\frac{du}{dt}=e^t$	$u=u(t)$	$u(t=0)=1$	$\tau=0{,}1$

15. Привести уравнения математической модели процесса к безразмерному виду. Определить неизвестные характерные параметры процесса. Обосновать целесообразность обезразмеривания.

15.1. $\dfrac{\partial c}{\partial t} = D\dfrac{\partial^2 c}{\partial x^2} - k\,c\,.$ Определить D_0, k_0.

15.2. $\dfrac{\partial c}{\partial t} = D\dfrac{\partial^2 c}{\partial x^2} - k\,c^2\,.$ Определить D_0, k_0.

15.3. $\dfrac{\partial c}{\partial t} + v\dfrac{\partial c}{\partial x} = -\,k\,c\,.$ Определить v_0, k_0.

15.4. $\dfrac{\partial c}{\partial t} + v\dfrac{\partial c}{\partial x} = -\,k\,c^2\,.$ Определить v_0, k_0.

15.5. $\dfrac{\partial c}{\partial t} + v\dfrac{\partial c}{\partial x} = D\dfrac{\partial^2 c}{\partial x^2} - k\,c\,.$ Определить v_0, D_0, k_0.

15.6. $\dfrac{\partial c}{\partial t} + v\dfrac{\partial c}{\partial x} = D\dfrac{\partial^2 c}{\partial x^2} - k\,c^2\,.$ Определить v_0, D_0, k_0.

15.7. $\dfrac{dc}{dt} = -\displaystyle\int_0^R \rho_2^0\, f\,\eta\, dr,\quad \dfrac{\partial f}{\partial t} + \eta\dfrac{\partial f}{dr} = 0.$ Определить η_0, f_0.

15.8. $\dfrac{\partial c}{\partial t} = D_L\,\dfrac{\partial^2 c}{\partial x^2} + D_R\,\dfrac{\partial^2 c}{\partial r^2} + \dfrac{D_R}{r}\dfrac{\partial c}{\partial r} - k\,c.$ Определить D_{L_0}, D_{R_0}, k_0.

15.9. $\dfrac{\partial c}{\partial t} + v\dfrac{\partial c}{\partial x} = D_L\,\dfrac{\partial^2 c}{\partial x^2} + D_R\,\dfrac{\partial^2 c}{\partial r^2} + \dfrac{D_R}{r}\dfrac{\partial c}{\partial r} - k\,c.$

Определить v_0, D_{L_0}, D_{R_0}, k_0.

15.10. $\dfrac{\partial c}{\partial t} + v\dfrac{\partial c}{\partial x} = D_L\,\dfrac{\partial^2 c}{\partial x^2} + D_R\,\dfrac{\partial^2 c}{\partial r^2} + \dfrac{D_R}{r}\dfrac{\partial c}{\partial r} - k\,c^2.$

Определить v_0, D_{L_0}, D_{R_0}, k_0.

16. Записать неявную разностную схему для системы уравнений.

16.1. $\dfrac{\partial u}{\partial t}=0.3\dfrac{\partial^2 u}{\partial x^2}+2uv$ $\quad$ $\dfrac{\partial v}{\partial t}=0.2\dfrac{\partial^2 v}{\partial x^2}-2uv$ $\quad$ $u=u(t,x)$, $v=v(t,x)$

16.2. $\dfrac{\partial u}{\partial t}=0.5\dfrac{\partial^2 u}{\partial x^2}-3u^2v$ $\quad$ $\dfrac{\partial v}{\partial t}=0.7\dfrac{\partial^2 v}{\partial x^2}+3u^2v$ $\quad$ $u=u(t,x)$, $v=v(t,x)$

16.3. $x\dfrac{\partial u}{\partial t}+x\dfrac{\partial u}{\partial x}=\dfrac{\partial^2 u}{\partial x^2}+v-x^2$ $\quad$ $t\dfrac{\partial v}{\partial t}+t\dfrac{\partial v}{\partial x}=u-t^2$ $\quad$ $u=u(t,x)$, $v=v(t,x)$

16.4. $\dfrac{\partial u}{\partial t}+\dfrac{\partial u}{\partial x}=\dfrac{\partial^2 u}{\partial x^2}+v$ $\quad$ $\dfrac{\partial v}{\partial t}=\dfrac{\partial v}{\partial x}+tx-u$ $\quad$ $u=u(t,x)$, $v=v(t,x)$

16.5. $2\dfrac{\partial u}{\partial t}+t\dfrac{\partial u}{\partial x}=\dfrac{\partial^2 u}{\partial x^2}+v-2t$ $\quad$ $x\dfrac{\partial v}{\partial t}+\dfrac{\partial v}{\partial x}=2u+2x$ $\quad$ $u=u(t,x)$, $v=v(t,x)$

16.6. $t\dfrac{\partial u}{\partial t}-2x\dfrac{\partial u}{\partial x}=2\dfrac{\partial^2 u}{\partial x^2}+v$ $\quad$ $t\dfrac{\partial v}{\partial t}=2x\dfrac{\partial v}{\partial x}+u$ $\quad$ $u=u(t,x)$, $v=v(t,x)$

16.7. $t\dfrac{\partial u}{\partial t}=\dfrac{\partial^2 u}{\partial x^2}+ve^t$ $\quad$ $e^t\dfrac{\partial v}{\partial t}-\dfrac{\partial v}{\partial x}=u-t$ $\quad$ $u=u(t,x)$, $v=v(t,x)$

16.8. $\dfrac{\partial u}{\partial t}=x\dfrac{\partial^2 u}{\partial x^2}+e^x(1-v)$ $\quad$ $\dfrac{\partial v}{\partial t}-e^x\dfrac{\partial v}{\partial x}=x-u$ $\quad$ $u=u(t,x)$, $v=v(t,x)$

16.9. $\dfrac{\partial u}{\partial t}+\dfrac{\partial u}{\partial x}=\dfrac{\partial^2 u}{\partial x^2}+v-t$ $\quad$ $x\dfrac{\partial v}{\partial t}+t\dfrac{\partial v}{\partial x}=u$ $\quad$ $u=u(t,x)$, $v=v(t,x)$

16.10. $x\left(\dfrac{\partial u}{\partial t}+\dfrac{\partial u}{\partial x}\right)=\dfrac{\partial^2 u}{\partial x^2}+v$ $\quad$ $\dfrac{\partial v}{\partial t}+t\dfrac{\partial v}{\partial x}=2u+t$ $\quad$ $u=u(t,x)$, $v=v(t,x)$

16.11. $x\dfrac{\partial u}{\partial t}=\dfrac{\partial^2 u}{\partial x^2}+v-2$ $\quad$ $\dfrac{\partial v}{\partial t}+x\dfrac{\partial v}{\partial x}=2x(u-x^2)$ $\quad$ $u=u(t,x)$, $v=v(t,x)$

16.12. $e^x\dfrac{\partial u}{\partial t}+e^t\dfrac{\partial u}{\partial x}=e^t\dfrac{\partial^2 u}{\partial x^2}+2v$ $\quad$ $\dfrac{\partial v}{\partial t}+te^{-x}\dfrac{\partial v}{\partial x}=u$ $\quad$ $u=u(t,x)$, $v=v(t,x)$

17. Выбрать метод численного решения для уравнений. Обосновать выбор.

17.1. $3\dfrac{d^2u}{dx^2}=2u+x^2;$ $\quad 3\dfrac{d^2u}{dx^2}=-2u+x^2;$ $\quad u = u(x).$

17.2. $8\dfrac{du}{dx}+0.7\dfrac{d^2u}{dx^2}+4u=0;$ $\quad 8\dfrac{du}{dx}+0.7\dfrac{d^2u}{dx^2}-4u=0;$ $\quad u = u(x).$

17.3. $0.2\dfrac{d^2u}{dx^2}=5\dfrac{du}{dx}-6ux;$ $\quad 0.2\dfrac{d^2u}{dx^2}=5\dfrac{du}{dx}+6ux;$ $\quad u = u(x).$

17.4. $7\dfrac{d^2u}{dx^2}=6u+x^2e^x;$ $\quad 7\dfrac{d^2u}{dx^2}=x^2e^x;$ $\quad u = u(x).$

17.5. $6\dfrac{du}{dx}+0.3\dfrac{d^2u}{dx^2}-5x=0;$ $\quad 6\dfrac{du}{dx}+0.3\dfrac{d^2u}{dx^2}-5x=u;$ $\quad u = u(x).$

17.6. $6\dfrac{du}{dx}+0.3\dfrac{d^2u}{dx^2}-5x=0;$ $\quad 6\dfrac{du}{dx}+0.3\dfrac{d^2u}{dx^2}-5x=u^2;$ $\quad u = u(x).$

17.7. $6\dfrac{du}{dx}+0.3\dfrac{d^2u}{dx^2}-5x=u;$ $\quad 6\dfrac{du}{dx}+0.3\dfrac{d^2u}{dx^2}-5x=u^2;$ $\quad u = u(x).$

17.8. $9\dfrac{du}{dx}+0.2\dfrac{d^2u}{dx^2}=u^2+2x;$ $\quad 9\dfrac{du}{dx}+0.2\dfrac{d^2u}{dx^2}=-u^2-2x;$ $\quad u = u(x).$

17.9. $\dfrac{du}{dx}=2\dfrac{d^2u}{dx^2}+7x;$ $\quad u = u(x).$

$\dfrac{\partial u}{\partial y}=2\dfrac{\partial^2u}{\partial x^2}+7x;$ $\quad u = u(x, y).$

17.10. $\dfrac{d^2u}{dx^2}=\dfrac{3x^2-2}{e^x};$ $\quad u = u(x).$

$2\dfrac{\partial^2u}{\partial x^2}+\dfrac{\partial^2u}{\partial y^2}=\dfrac{3x^2-2y^2}{e^{xy}};$ $\quad u = u(x, y).$

18. Привести уравнение к виду, удобному для использования метода простой итерации. Записать выражение для шага итерации. Записать итерационное соотношение, учитывая шаг итерации. Записать условие для окончания итерационного процесса. Выбрать начальное приближение.

18.1.	$2\dfrac{d^2u}{dx^2} = e^x(x-1)$	$u = u(x)$	$u(x=0)=0$	$u(x=1)=e$
18.2.	$2\dfrac{du}{dx} = \dfrac{d^2u}{dx^2} + 4x$	$u = u(x)$	$u(x=0)=0$	$u(x=1)=2$
18.3.	$\dfrac{du}{dx} + \dfrac{d^2u}{dx^2} + 2x = 0$	$u = u(x)$	$u(x=0)=0$	$u(x=1)=1$
18.4.	$\dfrac{du}{dx} = x\dfrac{d^2u}{dx^2}$	$u = u(x)$	$u(x=0)=2$	$u(x=1)=1$
18.5.	$\dfrac{du}{dx} + \dfrac{d^2u}{dx^2} = 3(2-x^2)$	$u = u(x)$	$u(x=0)=0$	$u(x=1)=2$
18.6.	$2\dfrac{du}{dx} = x\dfrac{d^2u}{dx^2} + 2$	$u = u(x)$	$u(x=0)=0$	$u(x=1)=2$
18.7.	$\dfrac{d^2u}{dx^2} = e^{-x}(2+x^2)$	$u = u(x)$	$u(x=0)=0$	$u(x=1)=\dfrac{1}{e}$
18.8.	$\dfrac{d^2u}{dx^2} = \dfrac{du}{dx} + 2xe^{x+1}$	$u = u(x)$	$u(x=0)=e$	$u(x=1)=0$
18.9.	$\dfrac{d^2u}{dx^2} + 2\dfrac{du}{dx} = 0$	$u = u(x)$	$u(x=0)=e^3$	$u(x=1)=e$
18.10.	$\dfrac{d^2u}{dx^2} - \dfrac{du}{dx} = e^x$	$u = u(x)$	$u(x=0)=0$	$u(x=1)=e$

19. Привести уравнение к виду, удобному для использования метода установления с применением неявной разностной схемы. Проверить сходимость прогонки. Записать итерационное соотношение. Найти α_1, β_1, u_N. Записать условие для окончания итерационного процесса. Выбрать начальное приближение.

19.1. $\dfrac{d^2u}{dx^2}-\dfrac{du}{dx}=e^x$ $\quad u=u(x)$ $\quad \dfrac{du}{dx}(x=0)=u(x=0)+1$

$\dfrac{du}{dx}(x=1)=2e$

19.2. $2\dfrac{du}{dx}=\dfrac{d^2u}{dx^2}+u$ $\quad u=u(x)$ $\quad \dfrac{du}{dx}(x=0)=1$

$\dfrac{du}{dx}(x=1)=2u(x=1)$

19.3. $\dfrac{du}{dx}+\dfrac{d^2u}{dx^2}+2x=0$ $\quad u=u(x)$ $\quad \dfrac{du}{dx}(x=0)=2$ $\quad \dfrac{du}{dx}(x=1)=0$

19.4. $-\dfrac{du}{dx}+x\dfrac{d^2u}{dx^2}=3x^2$ $\quad u=u(x)$ $\quad \dfrac{du}{dx}(x=0)=0$

$\dfrac{du}{dx}(x=1)=2u(x=1)+1$

19.5. $x\left(\dfrac{du}{dx}+4\right)=\dfrac{1}{3}\dfrac{d^2u}{dx^2}+u$ $\quad u=u(x)$ $\quad \dfrac{du}{dx}(x=0)=3u(x=0)+1$

$u(x=1)=2$

19.6. $x\left(\dfrac{du}{dx}-\dfrac{d^2u}{dx^2}\right)=2u$ $\quad u=u(x)$ $\quad \dfrac{du}{dx}(x=0)=4u(x=0)+2$

$\dfrac{du}{dx}(x=1)=0$

19.7. $\dfrac{d^2u}{dx^2}+5\dfrac{du}{dx}=3x(2-u)$ $\quad u=u(x)$ $\quad u(x=0)=1$

$\dfrac{du}{dx}(x=1)=3u(x=1)-2$

19.8. $3\dfrac{du}{dx}=4\dfrac{d^2u}{dx^2}-6(ux+2)$ $\quad u=u(x)$ $\quad \dfrac{du}{dx}(x=0)=3u(x=0)$

$$\frac{du}{dx}(x=1)=3u(x=1)$$

20. Привести уравнение к виду, удобному для использования метода установления с применением схемы Кранка–Николсона. Проверить сходимость прогонки. Записать итерационное соотношение. Найти α_1, β_1. Найти u_N. Записать условие для окончания итерационного процесса. Выбрать начальное приближение.

20.1. $\dfrac{d^2u}{dx^2}-\dfrac{du}{dx}=e^x$ $\qquad u=u(x)\qquad$ $\dfrac{du}{dx}(x=0)=u(x=0)+1$

$\dfrac{du}{dx}(x=1)=2e$

20.2. $2\dfrac{du}{dx}=\dfrac{d^2u}{dx^2}+u$ $\qquad u=u(x)\qquad$ $\dfrac{du}{dx}(x=0)=1$

$\dfrac{du}{dx}(x=1)=2u(x=1)$

20.3. $\dfrac{du}{dx}+\dfrac{d^2u}{dx^2}+2x=0$ $\qquad u=u(x)\qquad$ $\dfrac{du}{dx}(x=0)=2$

$\dfrac{du}{dx}(x=1)=0$

20.4. $-\dfrac{du}{dx}+x\dfrac{d^2u}{dx^2}=3x^2$ $\qquad u=u(x)\qquad$ $\dfrac{du}{dx}(x=0)=0$

$\dfrac{du}{dx}(x=1)=2u(x=1)+1$

20.5. $2\left(\dfrac{du}{dx}-1\right)=x\dfrac{d^2u}{dx^2}$ $\qquad u=u(x)\qquad$ $\dfrac{du}{dx}(x=0)=3u(x=0)+1$

$u(x=1)=2$

20.6. $\dfrac{du}{dx}-x\dfrac{d^2u}{dx^2}=2$ $\qquad u=u(x)\qquad$ $\dfrac{du}{dx}(x=0)=4u(x=0)+2$

$\dfrac{du}{dx}(x=1)=u(x=1)-1$

21. Привести уравнение к виду, удобному для использования метода простой итерации. Записать выражение для шага итерации. Записать итерационное соотношение, учитывая шаг итерации. Записать условие для окончания итерационного процесса. Выбрать начальное приближение.

21.1. $\frac{1}{6}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)=2x+1 \qquad u=u(x,y) \qquad \begin{cases} u(x=0,\,y)=3y^2 \\ u(x=1,\,y)=3y^2+2 \end{cases} \quad \begin{cases} u(x,\,y=0)=2x^3 \\ u(x,\,y=1)=2x^3+3 \end{cases}$

21.2. $0.5\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)=x+y \qquad u=u(x,y) \qquad \begin{cases} u(x=0,\,y)=0 \\ u(x=1,\,y)=y^2+y \end{cases} \quad \begin{cases} u(x,\,y=0)=0 \\ u(x,\,y=1)=x^2+x \end{cases}$

21.3. $1.5\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)=x+3 \qquad u=u(x,y) \qquad \begin{cases} u(x=0,\,y)=0 \\ u(x=1,\,y)=1+y^2/3 \end{cases} \quad \begin{cases} u(x,\,y=0)=x^2 \\ u(x,\,y=1)=x^2+x/3 \end{cases}$

21.4. $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=\frac{2x}{(y+1)^3} \qquad u=u(x,y) \qquad \begin{cases} u(x=0,\,y)=0 \\ u(x=1,\,y)=\frac{1}{y+1} \end{cases} \quad \begin{cases} u(x,\,y=0)=x \\ u(x,\,y=1)=0.5x \end{cases}$

21.5. $1.75\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)=7 \qquad u=u(x,y) \qquad \begin{cases} u(x=0,\,y)=y^2 \\ u(x=1,\,y)=(y-1)^2 \end{cases} \quad \begin{cases} u(x,\,y=0)=x^2 \\ u(x,\,y=1)=(x-1)^2 \end{cases}$

22. Привести уравнение к виду, удобному для использования метода установления с применением схемы расщепления. Для каждой из подсхем: проверить сходимость прогонки; записать итерационное соотношение; найти α_1, β_1; найти решение на правой границе. Записать условие для окончания итерационного процесса. Выбрать начальное приближение.

22.1. $$2y\frac{\partial^2 u}{\partial x^2}+3x\frac{\partial^2 u}{\partial y^2}+5y\frac{\partial u}{\partial y}=xy+4u \qquad u=u(x,y)$$

$$\begin{cases}\dfrac{\partial u}{\partial x}(x=0,\,y)=y\\ \dfrac{\partial u}{\partial x}(x=1,\,y)=u(x=1,\,y)\end{cases} \qquad \begin{cases}u(x,\,y=0)=0\\ u(x,\,y=1)=x\end{cases}$$

22.2. $$0.6\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)=3\frac{\partial u}{\partial x}+7u\,y \qquad u=u(x,y)$$

$$\begin{cases}u(x=0,\,y)=0\\ u(x=1,\,y)=y\end{cases} \qquad \begin{cases}\dfrac{\partial u}{\partial y}(x,\,y=0)=x\\ \dfrac{\partial u}{\partial y}(x,\,y=1)=u(x,\,y=1)\end{cases}$$

22.3. $$-4\frac{\partial u}{\partial y}+9\frac{\partial u}{\partial x}+0.8\frac{\partial^2 u}{\partial x^2}+1.3\frac{\partial^2 u}{\partial y^2}=7.1e^{x+y} \qquad u=u(x,y)$$

$$\begin{cases}\dfrac{\partial u}{\partial x}(x=0,\,y)=u(x=0,\,y)\\ u(x=1,\,y)=e^{y+1}\end{cases} \qquad \begin{cases}\dfrac{\partial u}{\partial y}(x,\,y=0)=u(x,\,y=0)\\ u(x,\,y=1)=e^{x+1}\end{cases}$$

22.4. $$2\frac{\partial^2 u}{\partial x^2}+7\frac{\partial^2 u}{\partial y^2}=5\left(\frac{\partial u}{\partial y}+\frac{\partial u}{\partial x}\right)+10(u+2xy-1) \qquad u=u(x,y)$$

$$\begin{cases}u(x=0,\,y)=y\\ u(x=1,\,y)=1-y\end{cases} \qquad \begin{cases}u(x,\,y=0)=x\\ u(x,\,y=1)=1-x\end{cases}$$

22.5. $0.3\frac{\partial^2 u}{\partial x^2}+0.5\frac{\partial^2 u}{\partial y^2}=7\frac{\partial u}{\partial x}-11\frac{\partial u}{\partial y}+xy(2u-1)$ $u=u(x,y)$

$$\begin{cases} u(x=0,\ y)=y \\ \frac{\partial u}{\partial x}(x=1,\ y)=y \end{cases} \qquad \begin{cases} u(x,\ y=0)=x \\ \frac{\partial u}{\partial y}(x,\ y=1)=x \end{cases}$$

22.6. $0.4\frac{\partial^2 u}{\partial x^2}+0.5\frac{\partial^2 u}{\partial y^2}+0.7\frac{\partial^2 u}{\partial z^2}+6\frac{\partial u}{\partial z}-9\frac{\partial u}{\partial y}=8y(xu+z)$ $u=u(x,y,z)$

$$\begin{cases} u(x=0,\ y,\ z)=zy \\ u(x=1,\ y,\ z)=z(y+1) \end{cases} \begin{cases} u(x,\ y=0,\ z)=zx \\ u(x,\ y=1,\ z)=z(x+1) \end{cases} \begin{cases} u(x,\ y,\ z=0)=0 \\ u(x,\ y,\ z=1)=x+y \end{cases}$$

22.7. $0.1\left(\frac{\partial^2 u}{\partial x^2}+2\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)=-3\frac{\partial u}{\partial y}+2\frac{\partial u}{\partial z}-5x(1-uyz)$ $u=u(x,y,z)$

$$\begin{cases} u(x=0,\ y,\ z)=0 \\ u(x=1,\ y,\ z)=yz \end{cases} \qquad \begin{cases} u(x,\ y=0,\ z)=0 \\ u(x,\ y=1,\ z)=xz \end{cases} \qquad \begin{cases} u(x,\ y,\ z=0)=0 \\ u(x,\ y,\ z=1)=xy \end{cases}$$

22.8. $3\frac{\partial^2 u}{\partial x^2}+5\frac{\partial^2 u}{\partial y^2}+7\frac{\partial^2 u}{\partial z^2}-8\frac{\partial u}{\partial z}=2\frac{\partial u}{\partial x}+9(u+yz-4xy)$ $u=u(x,y,z)$

$$\begin{cases} u(x=0,\ y,\ z)=-\,yz \\ u(x=1,\ y,\ z)=(4-z)y \end{cases} \begin{cases} u(x,\ y=0,\ z)=0 \\ u(x,\ y=1,\ z)=4x-z \end{cases} \begin{cases} u(x,\ y,\ z=0)=4xy \\ u(x,\ y,\ z=1)=4xy-y \end{cases}$$

22.9. $2\frac{\partial u}{\partial x}+3\frac{\partial^2 u}{\partial x^2}+8\frac{\partial^2 u}{\partial y^2}+11\frac{\partial^2 u}{\partial z^2}=5\frac{\partial u}{\partial y}+3z(xy+u)$ $u=u(x,y,z)$

$$\begin{cases} u(x=0,\ y,\ z)=0 \\ u(x=1,\ y,\ z)=yz \end{cases} \qquad \begin{cases} u(x,\ y=0,\ z)=0 \\ u(x,\ y=1,\ z)=xz \end{cases} \qquad \begin{cases} u(x,\ y,\ z=0)=0 \\ u(x,\ y,\ z=1)=xy \end{cases}$$

22.10. $0.2\frac{\partial^2 u}{\partial x^2}+0.5\frac{\partial^2 u}{\partial y^2}+1.3\frac{\partial^2 u}{\partial z^2}=2\frac{\partial u}{\partial x}-2zy$ $u=u(x,y,z)$

$$\begin{cases} u(x=0,\ y,\ z)=0 \\ u(x=1,\ y,\ z)=yz \end{cases} \qquad \begin{cases} u(x,\ y=0,\ z)=0 \\ u(x,\ y=1,\ z)=xz \end{cases} \qquad \begin{cases} u(x,\ y,\ z=0)=0 \\ u(x,\ y,\ z=1)=xy \end{cases}$$

23. Привести уравнение к виду, удобному для использования метода установления с применением схемы переменных направлений. Для каждой из подсхем: проверить сходимость прогонки; записать итерационное соотношение; найти α_1, β_1; найти решение на правой границе. Записать условие для окончания итерационного процесса. Выбрать начальное приближение.

23.1. $$2y\frac{\partial^2 u}{\partial x^2}+3x\frac{\partial^2 u}{\partial y^2}+5y\frac{\partial u}{\partial y}=xy+4u \qquad u=u(x,y)$$

$$\begin{cases} u(x=0,\, y)=0 \\ u(x=1,\, y)=y \end{cases} \qquad \begin{cases} \dfrac{\partial u}{\partial y}(x,\, y=0)=x \\ \dfrac{\partial u}{\partial y}(x,\, y=1)=u(x,\, y=1) \end{cases}$$

23.2. $$-\frac{\partial u}{\partial y}+5\frac{\partial u}{\partial x}+0{,}8\frac{\partial^2 u}{\partial x^2}+1{,}2\frac{\partial^2 u}{\partial y^2}=7u \qquad u=u(x,y)$$

$$\begin{cases} \dfrac{\partial u}{\partial x}(x=0,\, y)=u(x=0,\, y) \\ \dfrac{\partial u}{\partial x}(x=1,\, y)=u(x=1,\, y) \end{cases} \qquad \begin{cases} u(x,\, y=0)=e^{x} \\ u(x,\, y=1)=e^{x+1} \end{cases}$$

23.3. $$2\frac{\partial^2 u}{\partial x^2}+3\frac{\partial^2 u}{\partial y^2}=\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+2(u+2xy-1) \qquad u=u(x,y)$$

$$\begin{cases} \dfrac{\partial u}{\partial x}(x=0,\, y)=1-2y \\ u(x=1,\, y)=1-y \end{cases} \qquad \begin{cases} \dfrac{\partial u}{\partial y}(x,\, y=0)=1-2x \\ u(x,\, y=1)=1-x \end{cases}$$

23.4. $$4\frac{\partial^2 u}{\partial x^2}+7\frac{\partial^2 u}{\partial y^2}=2\frac{\partial u}{\partial x}-3\frac{\partial u}{\partial y}+6x(u-3x-2y) \qquad u=u(x,y)$$

$$\begin{cases} \dfrac{\partial u}{\partial x}(x=0,\, y)=3 \\ \dfrac{\partial u}{\partial x}(x=1,\, y)=3 \end{cases} \qquad \begin{cases} \dfrac{\partial u}{\partial y}(x,\, y=0)=2 \\ \dfrac{\partial u}{\partial y}(x,\, y=1)=2 \end{cases}$$

23.5. $$5\frac{\partial^2 u}{\partial x^2}+6\frac{\partial^2 u}{\partial y^2}-2xy\frac{\partial u}{\partial x}+2uy=0 \qquad u=u(x,y)$$

$$\begin{cases} u(x=0,\, y)=0 \\ \dfrac{\partial u}{\partial x}(x=1,\, y)=u(x=1,\, y) \end{cases} \qquad \begin{cases} 3\dfrac{\partial u}{\partial y}(x,\, y=0)=u(x,\, y=0) \\ 4\dfrac{\partial u}{\partial y}(x,\, y=1)=u(x,\, y=1) \end{cases}$$

24. Построить тестовую задачу для проверки устойчивости разностной схемы, аппроксимирующей уравнение.

24.1. $\dfrac{\partial u}{\partial t}+x\dfrac{\partial u}{\partial x}=e^t\,\dfrac{\partial^2 u}{\partial x^2}+2xe^t$ $\quad u=u(t,x)$, $\quad u(t=0,x)=x$ $\quad \begin{cases} u(t,x=0)=0 \\ \dfrac{\partial u}{\partial x}(t,x=1)=u(t,x=1) \end{cases}$

24.2. $2\dfrac{\partial u}{\partial t}=\dfrac{\partial u}{\partial x}+\dfrac{\partial^2 u}{\partial x^2}-2(u+e^x)$ $\quad u=u(t,x)$, $\quad u(t=0,x)=0$ $\quad \begin{cases} u(t,x=0)=t \\ \dfrac{\partial u}{\partial x}(t,x=1)=u(t,x=1) \end{cases}$

24.3. $\dfrac{\partial u}{\partial t}+3\dfrac{\partial u}{\partial x}=\dfrac{\partial^2 u}{\partial x^2}+e^x(2t+1)$ $\quad u=u(t,x)$, $\quad u(t=0,x)=0$ $\quad \begin{cases} u(t,x=0)=t \\ \dfrac{\partial u}{\partial x}(t,x=1)=u(t,x=1) \end{cases}$

24.4. $\dfrac{\partial c}{\partial t}=D\dfrac{\partial^2 c}{\partial x^2}-k\,c$ $\quad c=c(t,x)$, $\quad c(t=0,x)=c^0(x)$ $\quad \begin{cases} c(t,x=0)=c_1(t) \\ c(t,x=1)=c_2(t) \end{cases}$

24.5. $\dfrac{\partial c}{\partial t}=D\dfrac{\partial^2 c}{\partial x^2}-k\,c^2$ $\quad c=c(t,x)$, $\quad c(t=0,x)=c^0(x)$ $\quad \begin{cases} c(t,x=0)=c_1(t) \\ c(t,x=1)=c_2(t) \end{cases}$

24.6. $3\dfrac{\partial u}{\partial t}=2\dfrac{\partial^2 u}{\partial x^2}-3u$ $\quad u=u(t,x)$, $\quad u(t=0,x)=x$ $\quad \begin{cases} u(t,x=0)=t \\ \dfrac{\partial u}{\partial x}(t,x=1)=u(t,x=1) \end{cases}$

24.7. $\dfrac{\partial u}{\partial t}=2\left(\dfrac{\partial u}{\partial x}-u\right)+e^x$ $\quad u=u(t,x)$, $\quad u(t=0,x)=0$ $\quad u(t,x=1)=t\,e$

24.8. $2\dfrac{\partial u}{\partial t}+3\dfrac{\partial u}{\partial x}=u(2x+3t)$ $\quad u=u(t,x)$, $\quad u(t=0,x)=1$ $\quad u(t,x=0)=1$

24.9. $2\left(\dfrac{\partial u}{\partial t}-\dfrac{\partial u}{\partial x}+2t+1\right)=\dfrac{\partial^2 u}{\partial x^2}$ $\quad u=u(t,x)$, $\quad u(t=0,x)=x^2$

$$\begin{cases} u(t,x=0)=0 \\ \dfrac{\partial u}{\partial x}(t,x=1)=u(t,x=1) \end{cases}$$

25. Построить тестовую задачу для проверки устойчивости разностной схемы, аппроксимирующей систему уравнений.

25.1. $$\frac{\partial u}{\partial t}+\frac{\partial u}{\partial x}=\frac{\partial^2 u}{\partial x^2}+v \qquad u=u(t,x) \qquad u(t=0,x)=0 \qquad \begin{cases} u(t,x=0)=0 \\ u(t,x=1)=t \end{cases}$$

$$\frac{\partial v}{\partial t}=\frac{\partial v}{\partial x}+tx-u \qquad v=v(t,x) \qquad v(t=0,x)=x \qquad v(t,x=1)=t+1$$

25.2. $$\frac{\partial u}{\partial t}+\frac{\partial u}{\partial x}=\frac{\partial^2 u}{\partial x^2}+2(v-1) \qquad u=u(t,x) \qquad u(t=0,x)=x^2 \qquad \begin{cases} u(t,x=0)=t^2 \\ u(t,x=1)=t^2+1 \end{cases}$$

$$\frac{\partial v}{\partial t}=\frac{\partial v}{\partial x}+t^2+x^2-u \qquad v=v(t,x) \qquad v(t=0,x)=x \qquad v(t,x=1)=t+1$$

25.3. $$t\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}+ve^t \qquad u=u(t,x) \qquad u(t=0,x)=x \qquad \begin{cases} u(t,x=0)=0 \\ u(t,x=1)=e^t \end{cases}$$

$$e^t\frac{\partial v}{\partial t}-\frac{\partial v}{\partial x}=u-t \qquad v=v(t,x) \qquad v(t=0,x)=0 \qquad v(t,x=1)=t$$

25.4. $$\frac{\partial u}{\partial t}+\frac{\partial u}{\partial x}=\frac{\partial^2 u}{\partial x^2}+v-t \qquad u=u(t,x) \qquad u(t=0,x)=x \qquad \begin{cases} u(t,x=0)=t \\ u(t,x=1)=et+1 \end{cases}$$

$$x\frac{\partial v}{\partial t}+t\frac{\partial v}{\partial x}=u \qquad v=v(t,x) \qquad v(t=0,x)=e^x+1 \qquad v(t,x=0)=t+2$$

25.5. $$\frac{\partial u}{\partial t}=x\frac{\partial^2 u}{\partial x^2}+e^x(1-v) \qquad u=u(t,x) \qquad u(t=0,x)=0 \qquad \begin{cases} u(t,x=0)=t \\ u(t,x=1)=et \end{cases}$$

$$\frac{\partial v}{\partial t}-e^x\frac{\partial v}{\partial x}=x-u \qquad v=v(t,x) \qquad v(t=0,x)=0 \qquad v(t,x=1)=t$$

25.6. $$x\left(\frac{\partial u}{\partial t}+\frac{\partial u}{\partial x}\right)=\frac{\partial^2 u}{\partial x^2}+v \qquad u=u(t,x) \qquad u(t=0,x)=x \qquad \begin{cases} u(t,x=0)=t^2 \\ u(t,x=1)=t^2+1 \end{cases}$$

$$\frac{\partial v}{\partial t}+t\frac{\partial v}{\partial x}=2u+t \qquad v=v(t,x) \qquad v(t=0,x)=x \qquad v(t,x=0)=0$$

25.7. $2\dfrac{\partial u}{\partial t}+t\dfrac{\partial u}{\partial x}=\dfrac{\partial^2 u}{\partial x^2}+v-2t$ $\quad u=u(t,x)$ $\quad u(t=0,x)=x^2$ $\quad \begin{cases} u(t,x=0)=t \\ u(t,x=1)=t+1 \end{cases}$

$x\dfrac{\partial v}{\partial t}+\dfrac{\partial v}{\partial x}=2(u+x)$ $\quad v=v(t,x)$ $\quad v(t=0,x)=0$ $\quad v(t,x=0)=2t$

25.8. $x\left(\dfrac{\partial u}{\partial t}+\dfrac{\partial u}{\partial x}+x\right)=\dfrac{\partial^2 u}{\partial x^2}+v$ $\quad u=u(t,x)$ $\quad u(t=0,x)=0$ $\quad \begin{cases} u(t,x=0)=2t \\ u(t,x=1)=t \end{cases}$

$t\left(\dfrac{\partial v}{\partial t}+\dfrac{\partial v}{\partial x}+t\right)=u$ $\quad v=v(t,x)$ $\quad v(t=0,x)=2x$ $\quad v(t,x=0)=0$

25.9. $x\dfrac{\partial u}{\partial t}=\dfrac{\partial^2 u}{\partial x^2}+v-2$ $\quad u=u(t,x)$ $\quad u(t=0,x)=x^2+1$ $\quad \begin{cases} u(t,x=0)=e^t \\ u(t,x=1)=e^t+1 \end{cases}$

$\dfrac{\partial v}{\partial t}+x\dfrac{\partial v}{\partial x}=2x(u-x^2)$ $\quad v=v(t,x)$ $\quad v(t=0,x)=x$ $\quad v(t,x=0)=0$

25.10. $e^x\dfrac{\partial u}{\partial t}+e^t\dfrac{\partial u}{\partial x}=e^t\dfrac{\partial^2 u}{\partial x^2}+2v$ $\quad u=u(t,x)$ $\quad u(t=0,x)=e^x$ $\quad \begin{cases} u(t,x=0)=t^2+1 \\ u(t,x=1)=t^2+e \end{cases}$

$\dfrac{\partial v}{\partial t}+te^{-x}\dfrac{\partial v}{\partial x}=u$ $\quad v=v(t,x)$ $\quad v(t=0,x)=0$ $\quad v(t,x=0)=t$

25.11. $t\dfrac{\partial u}{\partial t}-2x\dfrac{\partial u}{\partial x}=2\dfrac{\partial^2 u}{\partial x^2}+v$ $\quad u=u(t,x)$ $\quad u(t=0,x)=0$ $\quad \begin{cases} u(t,x=0)=2t \\ u(t,x=1)=3t \end{cases}$

$t\dfrac{\partial v}{\partial t}=2x\dfrac{\partial v}{\partial x}+u$ $\quad v=v(t,x)$ $\quad v(t=0,x)=0$ $\quad v(t,x=1)=t$

25.12. $\dfrac{\partial u}{\partial t}=\dfrac{\partial^2 u}{\partial x^2}-v+t+e^t$ $\quad u=u(t,x)$ $\quad u(t=0,x)=e^x+1$ $\quad \begin{cases} u(t,x=0)=e^t+1 \\ u(t,x=1)=e^t+e \end{cases}$

$e^t\dfrac{\partial v}{\partial t}+\dfrac{\partial v}{\partial x}=u$ $\quad v=v(t,x)$ $\quad v(t=0,x)=e^x$ $\quad v(t,x=0)=t+1$

25.13. $\dfrac{\partial u}{\partial t}=\dfrac{\partial^2 u}{\partial x^2}+x(v-tx)$ $\quad u=u(t,x)$ $\quad u(t=0,x)=2x$ $\quad \begin{cases} u(t,x=0)=0 \\ u(t,x=1)=e^t+1 \end{cases}$

$x\dfrac{\partial v}{\partial t}+t\dfrac{\partial v}{\partial x}=u+x^2+t^2-x$ $\quad v=v(t,x)$ $\quad v(t=0,x)=1$ $\quad v(t,x=0)=e^t$